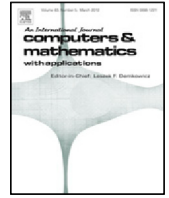




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Postprocessing Galerkin method using quadratic spline wavelets and its efficiency

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ABSTRACT

The wavelet-Galerkin method is a useful tool for solving differential equations mainly because the condition number of the stiffness matrix is independent of the matrix size and thus the number of iterations for solving the discrete problem by the conjugate gradient method is small. We have recently proposed a quadratic spline wavelet basis that has a small condition number and a short support. In this paper we use this basis in the Galerkin method for solving the second-order elliptic problems with Dirichlet boundary conditions in one and two dimensions and by an appropriate post-processing we achieve the L^2 -error of order $\mathcal{O}(h^4)$ and the H^1 -error of order $\mathcal{O}(h^3)$, where h is the step size. The rate of convergence is the same as the rate of convergence for the Galerkin method with cubic spline wavelets. We show theoretically as well as numerically that the presented method outperforms the Galerkin method with other quadratic or cubic spline wavelets. Furthermore, we present local post-processing for example of the equation with Dirac measure on the right-hand side.

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1. Introduction

Let $d \in \mathbb{N}$ and $\Omega = (0, 1)^d$. In this paper we focus on the case $d = 1$ and $d = 2$, but all that follows can be generalized. We use the wavelet-Galerkin method to solve the second order elliptic equation

$$-\sum_{i=1}^d \sum_{j=1}^d \frac{\partial}{\partial x_j} \left(a_{i,j}(x) \frac{\partial u}{\partial x_i} \right) + b(x)u = f \text{ on } \Omega, \quad u = 0 \text{ on } \partial\Omega. \quad (1)$$

We assume that $b(x) \geq B > 0$, the functions $a_{i,j}$, b and f are sufficiently smooth on $\overline{\Omega}$, and that $a_{i,j}$ satisfy the uniform ellipticity condition

$$\sum_{i=1}^d \sum_{j=1}^d a_{i,j}(x) x_i x_j \geq C \sum_{i=1}^d x_i^2, \quad x = (x_1, \dots, x_d), \quad (2)$$

where $C > 0$ is independent of x . The divergence form of Eq. (1) is

$$-\nabla \cdot (a \nabla u) + bu = f \text{ on } \Omega, \quad u = 0 \text{ on } \partial\Omega, \quad (3)$$

where ∇ denotes the standard nabla operator and the matrix a is a matrix of functions $a_{i,j}$. This equation is a basic differential equation and it describes a wide variety of phenomena from physics and engineering. There is already a vast literature on

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