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A semi analytical method for the free vibration of doubly-curved shells of revolution

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ABSTRACT

In this paper, a semi analytical method is used to investigate the free vibration of doubly-curved shells of revolution with arbitrary boundary conditions. The doubly-curved shells of revolution are divided into their segments in the meridional direction, and the theoretical model for vibration analysis is formulated by applying Flügge's thin shell theory. Regardless of the boundary conditions, the displacement functions of shell segments are composed by the Jacobi polynomials along the revolution axis direction and the standard Fourier series along the circumferential direction. The boundary conditions at the ends of the doubly-curved shells of revolution and the continuous conditions at two adjacent segments were enforced by the penalty method. Then, the natural frequencies of the doubly-curved shells are obtained by using the Rayleigh–Ritz method. For arbitrary boundary conditions, this method does not require any changes to the mathematical model or the displacement functions, and it is very effective in the analysis of free vibration for doubly-curved shells of revolution. The credibility and exactness of proposed method are compared with the results of finite element method (FEM), and some numerical results are reported for free vibration of the doubly-curved shells of revolution under classical and elastic boundary conditions. Results of this paper can provide reference data for future studies in related field.

1. Introduction

The doubly-curved shells of revolution are widely used in aerospace, marine structures, civil and mechanical engineering. And the combinations of the doubly-curved paraboloidal, elliptical, circular toroidal, catenary, cycloidal and hyperbolical shells of revolution are often encountered in practical engineering applications. So it is very important to study the free vibration characteristics of the doubly-curved shells of revolution.

To emphasize the characteristics and the purpose of this paper, the existing literature for doubly-curved shells of revolution is reviewed and shown below: Leissa [1] and Qatu [2] analyzed not only the classical thin shell theories (Love's, Reissner's, Naghdi's, Sanders', and Flügge's, etc.) but also the thick shell theories, in addition the research on the vibration analysis of the shells was reviewed. Thin shell theories are based on the Kirchhoff–Love kinematic hypothesis that normal to the undeformed middle surface remains straight and normal to the deformed middle surface and suffer no extension. Shear deformation and rotary inertia factors should be considered in thicker shells. Based on these theories, researchers have proposed some analysis methods to analyse the free vibration for doubly-curved shells of revolution. Ding Zhou [3] studied

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the vibration analysis of the natural frequencies and mode shapes of a class of doubly-curved shells with different boundary conditions. Qatu et al. [4] presented vibration frequencies for spherical, cylindrical, hyperbolical and paraboloidal shells with 21 possible boundary conditions by using thin shallow shell theory and the Ritz method. Chen M et al. [5] presented an analytical method for analyzing the free and forced vibration characteristics of a ring-reinforced conical cylindrical shell with arbitrary boundary conditions, where the stiffener with a rectangular cross section is treated as a separate member and the equation of the annular plate is used to describe the movement of the stiffener. Jiang S et al. [6] derived a unified solution for the vibration analysis of doubly curved shallow shells with arbitrary elastic supports by using an improved trigonometric series and the Rayleigh–Ritz method. Tornabene et al. [7–11] studied the dynamic behavior of functionally graded or laminated composite doubly-curved shells and panels of revolution using the Generalized Differential Quadrature (GDQ) method. Tavakoli et al. [12] proposed a substructure synthesis method for vibration analysis of axisymmetric hermetic shell based on state space mathematics. Zhang C, Jin G et al. [13] investigated the vibration characteristics of circular cylindrical double-shell structures with different boundary conditions by using the improved Fourier series method based on Hamilton's principle. Cheng L et al. [14] analyzed the vibration of a circular cylindrical shell with a circular plate by using the spring stiffness technique and the variational method. Ghavanloo E et al. [15] presented vibration analysis of doubly-curved shallow shells, which are made of an orthotropic material. Mochida Y et al. [16] investigated the free vibration of doubly-curved shells of revolution using the Superposition-Galerkin method. Fadaee M et al. [17] proposed a new generic exact solution for free vibration of doubly-curved shallow shell panel, which used a set of potential functions and auxiliary variables to present an exact Levy-type closed-form solution for free vibration of a doubly curved FG shell panel. Liew K M et al. [18] studied the vibration of doubly-curved shallow shells. Wong et al. [19] investigated the low frequency vibration of the torsion free axisymmetric modes of thin cylindrical shell with hemispherical caps vibrating in vacuo using membrane approximation. Qu Y et al. [20–24] proposed a kind of domain decomposition method by a modified variational method, and studied the vibration characteristics of various combination shell structures consisting of conical, cylindrical, spherical shell and ring stiffener. In the method of domain decomposition, shell structures are divided into the shell components and their segments, the deflections of the segments are represented by Chebyshev polynomials and Fourier series, and the boundary conditions and continuous conditions of the structures are enforced by a modified variational principle and least-squares weighted residual method, the maximum energy equation of structures is written by using the Reissner–Naghdi's shell theory. Terry Hause [25] studied the problems related with the modeling and behavior in free vibration of doubly-curved sandwich panels with laminated face sheets. Tan [26] presented an efficient substructuring analysis method based on the first order shear deformation shell theory and the classical thin shell theory and investigated the free vibration characteristics for doubly-curved shells of revolution (an elliptic hyperboloid shell and a spherical shell, etc.). Naghsh [27] analyzed the free vibration of stringer stiffened shells with an arbitrary shape and a variable thickness using a meridional finite strip method. For both the circumferential and meridian directions of the shell element, Lagrange polynomials are used to interpolate the displacement variables. Stringer stiffeners are considered as the discrete curved beams. Al-Khatib O J et al. [28] studied the free vibration of a paraboloidal shell of revolution. Messina A [29] studied the free vibrations of multilayered doubly-curved shells of revolution based on a mixed variational approach and global piecewise-smooth functions. The research results on dynamic behavior of the composite laminated or the functionally graded material (FGM) doubly-curved shells and panels of revolution with arbitrary boundary conditions in references [30–35]. The displacement fields are represented by using several auxiliary functions and a standard Fourier series, and this method wonderfully eliminates all potential discontinuities of the admissible function and its derivatives at the edges. The classical boundary conditions and the general elastic restraint are imitated by the coupling spring stiffness technique, and then a unified numerical analysis model was presented to solve the free vibration of doubly-curved shells and panels of revolution by using the Rayleigh–Ritz method. Other studies related to the doubly-curved shells of revolution can be found in references [36–41]. Razavi S et al. [42] investigated the free vibration of a simply-supported MEE doubly-curved shell resting on a Pasternak foundation based on a shear deformation theory including the rotary inertia effect. The shell is assumed to be shallow and thin so that the in-plane electric and magnetic fields can be ignored. Grigorenko Ya. M et al. [43–46] used efficient numerical–analytical approaches to study the propagation of elastic waves in anisotropic inhomogeneous cylinders with circular and noncircular cross sections and the natural vibrations of anisotropic solid and hollow cylinders of finite length with different end conditions and so on. Bacciocchi M et al. [47] perform the free vibration analysis of several laminated composite doubly-curved shells, singly-curved shells and plates, characterized by a continuous thickness variation. Tornabene F et al. [48] presented a new numerical method to solve the weak formulation of the governing equations for the free vibrations of laminated composite shell structures with variable radii of curvature. Jiang S et al. [49] focused on the free linear vibrations of doubly-curved shallow reinforced shells of revolution by any number of beams of arbitrary lengths.

From the above literature review, we can know that most of existing literatures are focused on the vibration analysis of spherical and cylindrical shells. And some existing works put attention on the vibration analysis of doubly-curved shell of revolution by the Rayleigh–Ritz method, Generalized Differential Quadrature (GDQ) method, Fourier series method, domain decomposition method, or numerical method under the classical or elastic boundary condition. Thus, a unified, efficient and accurate formulation is necessary and of great significance to establish to analyze the free vibration of doubly-curved shells of revolution under arbitrary boundary conditions. The aim of this paper is to present a generalized and unified Jacobi–Ritz formulation to investigate the free vibration of doubly-curved paraboloidal, elliptical, and hyperbolical shells of revolution under arbitrary boundary conditions. The other purpose of this paper is to provide some useful results of the free vibration of some doubly-curved shells of revolution, which may be used as benchmarks in future research.