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The finite cell method for nearly incompressible finite strain plasticity problems with complex geometries

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ABSTRACT

In this paper, the performance of the Finite Cell Method is studied for nearly incompressible finite strain plasticity problems. The Finite Cell Method is a combination of the fictitious domain approach with the high-order Finite Element Method. It provides easy mesh generation capabilities for highly complex geometries; moreover, this method offers high convergence rates, the possibility to overcome locking and robustness against high mesh distortions. The performance of this method is numerically investigated based on computations of benchmark and applied problems. The results are also verified with the *h*- and *p*-version Finite Element Method. It is demonstrated that the Finite Cell Method is an appropriate simulation tool for large plastic deformations of structures with complex geometries and microstructured materials, such as porous and cellular metals that are made up of ductile materials obeying nearly incompressible J_2 theory of plasticity.

1. Introduction

The Finite Cell Method (FCM) is a fictitious domain approach based on the high-order Finite Element Method (FEM) in order to simplify the meshing of highly complex geometries [1,2]. The standard FEM discretizes the solution field using a geometry-conforming mesh. However, it can also be discretized independently using a simpler non-conforming mesh. As stated in the FCM Refs. [1–4], this approach has been followed in the scope of meshless and element-free methods, the generalized FEM, the extended FEM, and immersed boundary or fictitious domain methods. The FCM, due to its fictitious domain approach, can easily operate with almost any complex geometric model, ranging from boundary representations to voxel representations. Therefore, the FCM provides the possibility of an efficient integration of geometric modeling and FEM computations, which is a significant help when trying to reduce the total time of numerical simulations. There are also other methods trying to integrate geometric modeling and FEM computations. Examples are Isogeometric Analysis (IGA) with NURBS-based shape functions [5] and the high-order FEM with CAD-based blending functions for geometrical mappings [6,7].

The FCM applies a high-order basis, e.g. the hierarchical (integrated Legendre) basis of the *p*-FEM [1,2] as well as smooth B-spline and NURBS basis functions [8–10]. Therefore, the FCM inherits the advantages of high-order basis functions. The *p*-FEM basis functions [7,11,12] are used in the current research. The *p*-FEM yields high convergence rates provided that the solution is smooth enough or a proper mesh design is combined with the *p*-refinement [11,13–15]. It can overcome

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volumetric locking in nearly incompressible materials and thin-walled structures [11,13]. In addition, it is highly insensitive to element distortions [13]. Similar advantages can also be provided with IGA [5,16]. The sum of these outstanding features has recently led to extensions of high-order approaches such as IGA [17,18] and the p-FEM to the finite strain J_2 plasticity, which is characterized with nearly incompressible behavior. Considering ductile metals, the plastic deformation is purely incompressible; while, the elastic deformation is compressible and small. In the finite strain range, the large plastic deformation dominates the total deformation and makes it nearly incompressible [19,20].

In this paper, the FCM is extended to the finite strain J_2 plasticity. The FCM provides easy mesh generation capabilities due to its fictitious domain approach. Also, it provides a high convergence rate, a possibility to overcome locking behavior and robustness against high distortions stemming from its high-order basis. The J_2 theory with FCM is applicable in the finite strain elastoplastic analysis of a wide variety of engineering structures with complex geometries and microstructured materials which are made up of ductile metals. Typical examples are metal forming problems as well as the plasticity of porous and cellular materials. The FCM was first introduced for linear elastic problems [1,2]. Later, it was successfully extended to various fields in structural mechanics [3,4], including finite strain elasticity [10,21], small strain elastoplasticity [22,23] and damage mechanics [24,25]. Further extensions include topology optimization, biomedical engineering, numerical homogenization, wave propagation in heterogeneous materials, adaptive mesh-refinement and local enrichment, convection diffusion problems, thin-walled structures, iso-geometric-analysis, and multi-physical applications [3,4].

The outline of this paper is as follows: Section 2 summarizes the hyperelastic-based finite strain J_2 plasticity theory. In Section 3, we will briefly explain the FCM formulation for the nonlinear solution of finite strain elastoplastic problems. In Section 4, we will numerically investigate the performance of the FCM by computing benchmark problems as well as the applied problems. Finally, Section 5 will summarize the most important results and give an outlook to future work.

2. Finite strain J_2 plasticity

In the following, the finite strain plasticity of von Mises or J_2 plasticity theory with isotropic hardening is briefly summarized [19]. This model is formulated in spatial form with hyperelastic description of the elastic behavior. The main kinematic hypothesis is the local multiplicative decomposition of the deformation gradient, $\boldsymbol{F} = \frac{\partial \boldsymbol{x}}{\partial \boldsymbol{X}}$, into elastic $\boldsymbol{F^e}$ and plastic $\boldsymbol{F^p}$ contributions:

$$\boldsymbol{F} = \boldsymbol{F}^e \boldsymbol{F}^p. \tag{1}$$

For the elastic part of the deformation, the specific free energy function of the Hencky material is assumed, which yields the following simple constitutive equation for the Kirchhoff stress tensor

$$\boldsymbol{\tau} = \boldsymbol{D} : \boldsymbol{\varepsilon}^e \quad , \quad \boldsymbol{\varepsilon}^e = \frac{1}{2} \ln \boldsymbol{B^e} \tag{2}$$

where $\boldsymbol{\varepsilon}^e$ is the spatial logarithmic elastic strain tensor, $\boldsymbol{B^e} = \boldsymbol{F}^e (\boldsymbol{F}^e)^T$ is the elastic left Cauchy–Green tensor, and $\boldsymbol{D}$ has the format of the small strain isotropic elasticity tensor.

The classical von Mises yield function is formulated in terms of the deviatoric Kirchhoff stress tensor $\boldsymbol{s}$

$$\Phi(\boldsymbol{s}, \sigma_y) = \sqrt{\frac{3}{2} \boldsymbol{s} : \boldsymbol{s}} - \sigma_y(\alpha) \tag{3}$$

where $s = \boldsymbol{\tau} - \frac{1}{3} \mathrm{tr}\, \boldsymbol{\tau}$, σ_y is the uniaxial yield strength as a generally nonlinear function of the isotropic hardening parameter α.

The associative Prandtl–Reuss flow rule for ductile metals in the finite strain range reads:

$$\dot{\boldsymbol{F}}^p (\boldsymbol{F}^p)^{-1} = \sqrt{\frac{3}{2}} \dot{\gamma} \, (\boldsymbol{R}^e)^T \boldsymbol{n} \boldsymbol{R}^e \quad , \quad \boldsymbol{n} = \frac{\boldsymbol{s}}{\|\boldsymbol{s}\|} \tag{4}$$

where $\dot{\gamma}$ is the non-negative plastic multiplier and $\boldsymbol{R^e}$ is the elastic rotation tensor. This equation gives the evolution of the plastic deformation gradient. In addition, it implicates the incompressibility of the plastic flow [19], i.e. $\det \boldsymbol{F}^p = 1$ or $\mathrm{tr}\, \boldsymbol{L}^p = 0$ where $\boldsymbol{L}^p = \dot{\boldsymbol{F}}^p (\boldsymbol{F}^p)^{-1}$.

The isotropic hardening relation is the same as in the small strain theory,

$$\dot{\alpha} = \dot{\gamma}. \tag{5}$$

The hardening law and the flow rule are complemented by the Kuhn–Tucker loading/unloading conditions

$$\Phi \leq 0 \quad , \quad \dot{\gamma} \geq 0 \quad , \quad \Phi \, \dot{\gamma} = 0. \tag{6}$$

The aforementioned constitutive relations are integrated according to the backward or fully implicit Euler procedure in order to update the state variables, as explained in Appendix. The exponential mapping technique is also utilized for the flow rule in order to maintain its volume-preserving feature.

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