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Rational and semi-rational solutions of the y-nonlocal Davey–Stewartson I equation

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ABSTRACT

The nonlocal Davey–Stewartson (DS) I equation with a parity-time-symmetric potential with respect to the y-direction, which is called the y-nonlocal DS I equation, is a two-dimensional analogue of the nonlocal nonlinear Schrödinger (NLS) equation. The multi-breather solutions for the y-nonlocal DS I equation are derived by using the Hirota bilinear method. Lump-type solutions and hybrid solutions consisting of lumps sitting on periodic line waves are generated by long wave limits of the obtained soliton solutions. Also, various types of analytical solutions for the nonlocal NLS equation with negative nonlinearity, including both the Akhmediev breathers and the Peregrine rogue waves sitting on periodic line waves, can be generated with appropriate constraints on the parameters of the obtained exact solutions of the y-nonlocal DS I equation. Particularly, we show that a family of hybrid solitons describing the Peregrine rogue wave that coexists with the Akhmediev breather, both of them sitting on a spatially-periodic background can be thus obtained.

1. Introduction

Two decades ago, Bender and Boettcher [1] proposed the concept of space–time reflection symmetry, the so-called *parity-time (PT) symmetry* in quantum mechanics, which was found to be responsible for purely real spectra of non-Hermitian operators. They proved that wide classes of non-Hermitian Hamiltonians with PT symmetry have entirely real spectra. The seminal work by Bender and Boettcher [1] motivated a vast activity in the study of PT symmetry in quantum mechanics [2–5]. Optics and photonics have provided an excellent test bed for observing the phenomenon of PT symmetry in specially designed optical waveguide structures, in optical lattices and in other relevant physical settings [6–14]. However, during the past few years a series of studies [15–21] showed that if the complex-valued potential is not fully PT symmetric but is partially PT symmetric, it may also have new and interesting applications in optics and photonics and in other areas. Yang [15] introduced multidimensional complex optical potentials with partial PT symmetry. It is well known that the standard PT symmetry requires that the complex-valued external potential is invariant under complex conjugation and simultaneous reflection in all spatial coordinates. By using analytical and numerical techniques, Yang [15] has demonstrated that if the external potential is only partially PT symmetric, i.e., it is invariant under complex conjugation and reflection in a single spatial coordinate, then it can also possess all real eigenvalues and continuous families of solitons. Thus further studies in the area of partially PT-symmetric physical systems are both worthwhile and necessary.

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The many results in the area of one-dimensional PT-symmetric and partially PT-symmetric physical systems inspire naturally the extension of these studies to the more complicated multi-dimensional dynamical systems. Very recently, Fokas [22] introduced a two-dimensional nonlocal nonlinear Schrödinger (NLS) equation

$$\begin{aligned} &iA_t = A_{xx} + A_{yy} + (\epsilon V - 2Q)A, \\ &Q_{xx} - Q_{yy} = (\epsilon V)_{xx}, \end{aligned} \tag{1}$$

by using a certain reduction of a multi-dimensional system. Here

$$V = A(x, y, t)\,[A(x, -y, t)]^*, \epsilon = \pm 1, \tag{2}$$

the asterisk denotes complex conjugation, and A is a complex-valued function of real variables x, y, and t, and Q is a function of x, y, and t. This equation has a PT-symmetric potential in the y-direction, thus from now on we call it the y-nonlocal Davey–Stewartson I (DS I) equation. In particular, if we set $V = A(x, y, t)[A(x, y, t)]^*$ in Eq. (1), then it reduces to the usual DS I equation. Thus the y-nonlocal DS I equation is a partially PT-symmetric extension of the DS I equation. Other nonlocal extensions of the DS I equation were also introduced in Ref. [23]. Also, it is worth mentioning that the problem of generation of rogue waves was recently investigated in a class of nonlocal nonlinear NLS equations by Horikis and Ablowitz [24].

The y-nonlocal DS I equation is also a two-dimensional extension of the nonlocal NLS equation proposed by Ablowitz and Musslimani [25]:

$$q_t(x, t) - iq_{xx}(x, t) \pm 2iV(x, t)q(x, t) = 0\,,\, V(x, t) = q(x, t)q^*(-x, t). \tag{3}$$

The exact soliton and rational solutions of the nonlocal NLS equation were obtained by different methods [25–30]. However, for both the nonlocal NLS equation and the y-nonlocal DS I equation, different kinds of *rogue waves* coexisting with breathers and sitting on a background constituted by periodic line waves have not been studied before, to the best of our knowledge. It is worth mentioning that rogue waves, a term originally coined to provide a vivid description of the mysterious monstrous ocean waves [31], have recently attracted much attention in the study of their fundamental origin and complex dynamics in different physical systems [32–50]. The most recent theoretical and experimental results in this fast developing area were summarized in Refs. [51–54].

In a recent work [55], $(2+1)$-dimensional breather, rational, and semi-rational exact solutions of partially PT-symmetric nonlocal DS equations (of type I and type II) with respect to the x variable (the so-called x-nonlocal DS equations of type I and type II) have been reported. Also, exact $(2 + 1)$-dimensional N-soliton solutions and periodic line waves for the fully PT-symmetric nonlocal DS II equation were reported by employing the Hirota's bilinear method [55].

In this paper, we report various types of exact solutions for the y-nonlocal DS I equation, including breather and lump solutions, and hybrid solutions consisting of breathers and lumps sitting on periodic line wave background. The organization of the article is settled as follows. In Section 2, we discuss the analytical solutions and breathers of the y-nonlocal DS I equation. In Section 3, for different sets of parameter constraints of the obtained rational solutions, the lump solutions of y-nonlocal DS I equation (1) are obtained, and their typical features are analyzed and illustrated graphically. In Section 4, semi-rational solutions of the nonlocal DS I equation are derived and discussed. In Section 5, semi-rational solutions for the nonlocal NLS equation are given, namely the Peregrine rogue wave sitting on a spatially-periodic background, the Akhmediev breather sitting on the same type of background, and a complex waveform composed by the Peregrine rogue wave and the Akhmediev breather sitting on a spatially-periodic background. Section 6 contains a summary and a brief discussion of the obtained results.

2. Solitons and Breathers of the y-nonlocal DS I equation

In this section, we pay attention to the solutions of y-nonlocal DS I equation (1) by bilinear method. The y-nonlocal DS I equation is translated into the following bilinear form

$$\begin{aligned} &(D_x^2 + D_y^2 - iD_t)g \cdot f = 0, \\ &(D_x^2 - D_y^2)f \cdot f = 2\epsilon(f^2 - gh), \end{aligned} \tag{4}$$

and a dependent variable transformation in rational form is implemented:

$$A = \sqrt{2}\frac{g}{f}, Q = \epsilon - 2(\log f)_{xx}. \tag{5}$$

As three independent variables, f, g, h, are introduced to solve for one unknown A, one can impose without loss of generality that

$$[f(x, y, t)]^* = f(x, -y, t), [g(x, y, t)]^* = h(x, -y, t), \tag{6}$$

where D is a Hirota's bilinear differential operator [56] defined by $P(D_x, D_y, D_t,)F(x, y, t \cdots) \cdot G(x, y, t, \cdots) = P(\partial_x - \partial_{x'}, \partial_y - \partial_{y'}, \partial_t - \partial_{t'}, \cdots)F(x, y, t, \cdots)G(x', y', t', \cdots)|_{x'=x, y'=y, t'=t}$, and P is a polynomial of $D_x, D_y, D_t, \cdots$.