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A symbolic computation approach to constructing rogue waves with a controllable center in the nonlinear systems

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ABSTRACT

A symbolic computation approach to constructing higher order rogue waves with a controllable center of the nonlinear systems is presented, making use of their Hirota bilinear forms. As some examples, it turns out that some higher order rogue wave solutions of the Kadomtsev–Petviashvili (KP) type equations in (3+1) and (2+1)-dimensions are obtained. Some features of controllable center of rogue waves are graphically discussed.

1. Introduction

The rogue wave is thought of as an isolated huge wave with amplitude much larger than the average wave crests around it in the ocean [1,2]. In contrast to tsunamis and storms associated with typhoons that can be predicted hours (sometimes days) in advance, the particular danger of oceanic rogue waves is their sudden appearance as ″ waves that appear from nowhere and disappear without a trace ″ [3] only seconds before they hit a ship.

In mathematical physics, rogue wave solution is a kind of interesting rational solution and is localized both in space and time, which draws a big attention of mathematicians and physicists worldwide and such rational solutions describe significant nonlinear wave phenomena, particularly in oceanography [4,5], optical fibers [6,7], water wave tank [8], Bose–Einstein condensates [9], financial markets [10] and other related fields [11,12]. These studies imply that the rogue waves are generic phenomena in nonlinear systems, and thus inspire us to look for the rogue wave solution and its properties and applications for the new systems in different fields. Recently [13–15], the first order rogue wave and rational solutions to some (3 + 1) and (2 + 1)-dimensional systems are constructed by the symbolic computation approach. However, these forms of the higher order rogue wave solution with a controllable center have not been given before. It is inspired by the literatures [16,17], we provide a symbolic computation approach for constructing the higher order rogue waves with a controllable center of the higher dimensional nonlinear systems.

We would like to study the (2 + 1)-dimensional KP equation [18–22]

$$(u_t + 6uu_x + u_{xxx})_x + 3\sigma u_{yy} = 0, \tag{1}$$

where $\sigma = \pm 1$, as the prototype example. The KP equation (1), which may be thought of as a two spatial dimensional analog of the KdV equation

$$u_t + 6uu_x + u_{xxx} = 0, \tag{2}$$

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and plays the role in 2+1 dimensions that the KdV equation (2) plays in 1+1 dimensions. It is one of the classical prototype problems in the field of exactly solvable equations and arises generically in physical contexts with both choices of the sign of σ. It is classified as the KPI equation when $\sigma = -1$ and the KPII equation when $\sigma = 1$. For KPI equation, the solitons are unstable, whereas for KPII equation, they are stable. As we show early [20], for KPI equation, there exist periodic solitons (or breather/ complexiton), whilst for KPII equation, periodic solitons do not exist. With the help of Darboux transformation, the higher order rogue wave solutions of the NLS equation and their links with the KPI equation were studied by Dubard and Matveev [21]. Through symbolic computation with Maple, a general class of lump solutions of the KPI equation was presented by Ma [15], making use of Hirota bilinear form [22]. The first order rogue wave solution of the KPI equation was obtained by using extended homoclinic test approach (EHTA) [14] and Hirota bilinear form.

A (3 + 1)-dimensional generalized KP equation [23]

$$u_{xxxy} + 3(u_x u_y)_x + u_{tx} + u_{ty} - u_{zz} = 0, \tag{3}$$

will be also examined. Eq. (3) was investigated in [23] where Wronskian and Grammian formulations were established for Eq. (3). Multiple-soliton solutions and multiple singular soliton solutions of Eq. (3) was formally established by the simplified form of the Hirota's method [24]. It is natural and interesting to search for higher order rogue wave solutions to nonlinear wave equations, taking advantage of Hirota bilinear forms.

In this paper, we consider the (3+1) and (2+1)-dimensional rogue wave solution of the nonlinear systems with a symbolic computation approach. In Section 2, we propose the process of a symbolic computation approach for constructing higher order rogue wave solution. In Section 3, as the example, we derive higher order rogue wave solutions with the controllable center of the (3 + 1)-dimensional generalized KP equation. The higher order rogue wave solutions with the controllable center of the (2 + 1)-dimensional KP equation are also investigated in Section 4. The controllability of these solutions is shown through numerical simulation. A few concluding remarks will be given in Section 5.

2. A symbolic computation approach

We consider a (3 + 1)-dimensional nonlinear system in the form

$$N(u, u_t, u_x, u_y, u_z, u_{tt}, u_{tx}, u_{xy}, u_{xz} \cdots) = 0. \tag{4}$$

To determine $u(t, x, y, z)$ explicitly, we take the following steps:

Step 1: By Painlevè analysis, a transformation

$$u = T(f) \tag{5}$$

is made by using a dependent variable function f.

Step 2: By applying the transformation (5), the nonlinear system (4) can be converted into Hirota's bilinear form

$$G(D_\xi, D_z; f) = 0, \tag{6}$$

where $\xi = x + ay - ct$ and a, c are two real parameters. The D-operator [22] is defined by

$$D_\xi^m D_z^n f(\xi, z)g(\xi, z) = \left(\frac{\partial}{\partial \xi} - \frac{\partial}{\partial \xi'}\right)^m \left(\frac{\partial}{\partial z} - \frac{\partial}{\partial z'}\right)^n [f(\xi, z)g(\xi', z')]|_{\xi'=\xi, z'=z}.$$

Step 3: Assuming

$$f = \widetilde{F}_{n+1}(\xi, z; \alpha, \beta) = F_{n+1}(\xi, z) + 2\alpha z P_n(\xi, z) + 2\beta\xi Q_n(\xi, z) + (\alpha^2 + \beta^2)F_{n-1}(\xi, z), \tag{7}$$

with

$$F_n(\xi, z) = \sum_{k=0}^{n(n+1)/2} \sum_{i=0}^{k} a_{n(n+1)-2k,2i} z^{2i} \xi^{n(n+1)-2k},$$

$$P_n(\xi, z) = \sum_{k=0}^{n(n+1)/2} \sum_{i=0}^{k} b_{n(n+1)-2k,2i} z^{2i} \xi^{n(n+1)-2k},$$

$$Q_n(\xi, z) = \sum_{k=0}^{n(n+1)/2} \sum_{i=0}^{k} c_{n(n+1)-2k,2i} z^{2i} \xi^{n(n+1)-2k},$$

$F_0 = 1, F_{-1} = P_0 = Q_0 = 0$, where $a_{m,l}, b_{m,l}, c_{m,l}$ $(m, l \in \{0, 2, 4, \ldots, n(n+1)\}$ and α, β are real parameters. The coefficients $a_{m,l}, b_{m,l}, c_{m,l}$ can be determined, and arbitrary constants α, β are used to control the wave center.

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