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Finite fractal dimension of random attractor for stochastic non-autonomous strongly damped wave equation☆

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ABSTRACT

In this paper, we first prove the existence of a random attractor for stochastic non-autonomous strongly damped wave equations with additive white noise. Then we apply a criteria to obtain an upper bound of fractal dimension of the random attractor of considered system.

1. Introduction

Let $(\Omega, \mathcal{F}, \mathbb{P})$ be a probability space, where $\Omega = \{\omega \in C(\mathbb{R}, \mathbb{R}) : \omega(0) = 0\}$, the Borel σ-algebra $\mathcal{F}$ on Ω is generated by the compact open topology, and $\mathbb{P}$ is the corresponding Wiener measure on $\mathcal{F}$. For any $t \in \mathbb{R}$, define a mapping θ_t on Ω by $\theta_t\omega(\cdot) = \omega(t + \cdot) - \omega(t)$ for $\omega \in \Omega$, then $(\Omega, \mathcal{F}, \mathbb{P}, (\theta_t)_{t\in\mathbb{R}})$ is an ergodic metric dynamical system [1].

Consider the following initial boundary valued problem of non-autonomous strongly damped wave equations with additive white noise:

$$\begin{cases} u_{tt} + u_t - \Delta u - \alpha \Delta u_t + f(u, x) = g(x, t) + h(x)\dot{W}(t), t > \tau, x \in U, \tau \in \mathbb{R}, \\ u(x, t)|_{x\in\partial U} = 0, \quad t \geq \tau, \quad \tau \in \mathbb{R}, \\ u(x, \tau) = u_\tau(x), \quad u_t(x, \tau) = u_{1\tau}(x), \quad x \in U, \quad \tau \in \mathbb{R}, \end{cases} \tag{1.1}$$

where U is an open bounded set of $\mathbb{R}^n$ $(n \leq 3)$ with a smooth boundary ∂U; $\alpha > 0$ is the strong damping coefficient; $u(t) = u(x, t)$ is a real-valued function on $U \times [\tau, +\infty)$, $\tau \in \mathbb{R}$; $f(\cdot, x) \in C^1(\mathbb{R}, \mathbb{R})$; $h(\cdot) \in H_0^1(U) \cap H^2(U)$; $g(\cdot, t) \in C_b(\mathbb{R}, H_0^1(U))$, $C_b(\mathbb{R}, H_0^1(U))$ denotes the set of continuous bounded functions from $\mathbb{R}$ into $H_0^1(U)$; $W(t)$ is a two-sided real-valued Wiener process on the probability space $(\Omega, \mathcal{F}, \mathbb{P})$; the initial data $u_\tau(x), u_{1\tau}(x)$ are assumed to be independent of ω, but $u(t, \tau, \omega, x)$ and $u_t(t, \tau, \omega, x)$ depend on ω for $t > \tau$.

Eq. (1.1) can model a random perturbation of strongly damped wave equation. There have been a lot of profound results on the dynamics of a variety of systems related to Eq. (1.1). For example, the asymptotical behavior of solutions for deterministic strongly damped wave equation has been studied by many authors (see [2–15], etc.). For autonomous stochastic strongly damped wave equation, Wang and Zhou [16] have studied the asymptotical behavior of solutions (where g is independent of t).

Eq. (1.1) is a non-autonomous stochastic system where the external term g is time-dependent. For such a system, Wang established an efficacious theory about the existence of random attractors for corresponding non-autonomous random

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dynamical systems (called cocycle in [17]), see [17,18]. For the non-autonomous stochastic strongly damped wave equation on the unbounded domain, Wang and Zhou proved the existence of a random attractor, see [19]. Jones and Wang studied the asymptotic behavior of non-autonomous stochastic nonlinear wave equations with dispersive and dissipative terms defined on an unbounded domain, see [20].

The fractal dimension estimate of random attractors is an important problem (see [21,22]). The attractor has finite fractal dimension which means that the attractor can be mapped onto a compact subset of a finite dimensional Euclidean space. Recently, Zhou and Zhao in [23,24] gave some sufficient conditions to bound the fractal dimension of a random invariant set for a cocycle and applied these conditions to get the finiteness of fractal dimension of random attractor for non-autonomous stochastic damped wave equation with multiplicative white noise and additive white noise. Zhou and Tian et al. in [25] also gave similar sufficient conditions to bound the fractal dimension of a random invariant set for a cocycle and obtained the finiteness of fractal dimension of random attractor for stochastic reaction–diffusion equations with multiplicative white noise and additive white noise.

So far as we know, there were no results about the boundedness of fractal dimensions of random attractor of non-autonomous stochastic strongly damped wave equations. In this paper, motivated by the idea of [17,18,23–26], by carefully splitting the positivity of the linear operator in the corresponding random evolution equation of the first order in time, and by carefully decomposing the solutions of the system, we first prove the cocycle associated with non-autonomous stochastic strongly damped wave equations (1.1) has a random attractor in $H_0^1(U)\times L^2(U)$ which is bounded in $[H^2(U)\cap H_0^1(U)]\times H_0^1(U)$ through a recurrence method, then we apply a criteria to get an upper bound of fractal dimension of the random attractor of considered system.

This paper is organized as follows. In Section 2, we present some mathematical setting for our system. We first transfer the stochastic differential equation (1.1) into an equivalent random differential equation (2.5), then we show that the solutions mapping for this random equation (2.5) generate a continuous cocycle. In Section 3, we estimate the solutions of Eq. (2.5). We first consider the concrete bounds of the solutions, then we decompose the solutions of Eq. (2.5) into two parts. In Section 4, we obtain an upper bound of fractal dimension of random attractor for our system (1.1). And in Section 5, we give some remarks.

In this paper, the letters c_i $(i\in\mathbb{N})$ below are generic positive constants which do not depend on ω, τ and t.

2. Mathematical setting

Let $A=-\Delta$, then A is a self-adjoint positive linear sectorial operator with eigenvalues $\{\lambda_i\}_{i\in N}$: $0<\lambda_1\le\lambda_2\le\cdots\le\lambda_m\le\cdots$, $\lambda_m\to+\infty$ as $m\to+\infty$.

Define the rth powers A^r of A for $r\in\mathbb{R}$. Let $D(A^r)$ be a Hilbert space with inner product $(u,v)_{2r}=(A^ru,A^rv)$, and $E^r=D(A^{r+\frac12})\times D(A^r)$ for $r\in\mathbb{R}$. Denote the inner and norm of $L^2(U)$ as $(\cdot,\cdot)$ and $\|\cdot\|$, respectively. Write $E=H_0^1(U)\times L^2(U)$, and introduce a new weight inner product and norm in the Hilbert space E,

$$(\varphi_1,\varphi_2)_E=\mu(A^{\frac12}u_1,A^{\frac12}u_2)+(v_1,v_2)=\mu(u_1,u_2)_1+(v_1,v_2),\quad \|\varphi\|_E=(\varphi,\varphi)_E^{\frac12},\tag{2.1}$$

for any $\varphi_1=(u_1,v_1)^\top$, $\varphi_2=(u_2,v_2)^\top$, $\varphi\in E$, where μ is chosen as

$$\mu=\frac{4+(\alpha\lambda_1+1)\alpha+\frac{1}{\lambda_1}}{4+2(\alpha\lambda_1+1)\alpha+\frac{1}{\lambda_1}}\in\Big(\frac12,1\Big).\tag{2.2}$$

Clearly, the norm $\|\cdot\|_E$ in (2.1) is equivalent to the usual norm $\|\cdot\|_{H_0^1\times L^2}$ of E.

In the following, we convert the problem (1.1) into a random system without noise terms. Identify $\omega(t)$ with $W(t)$, i.e., $\omega(t)=W(t)$, $t\in\mathbb{R}$, and let $z(\theta_t\omega):=-\int_{-\infty}^0 e^s(\theta_t\omega)(s)ds$ $(t\in\mathbb{R})$ be an Ornstein–Uhlenbeck stationary process which solves the Itô equation $dz+zdt=dW(t)$. It is known from [1,27] that for almost every $\omega\in\Omega$, $t\mapsto z(\theta_t\omega)$ is continuous in t and

$$\lim_{t\to+\infty}e^{-\gamma t}|z(\theta_{-t}\omega)|=0,\quad \forall\gamma>0;\quad \mathbf{E}[|z(\theta_t\omega)|^r]=\frac{\Gamma(\frac{1+r}{2})}{\sqrt{\pi}},\quad \forall r>0,t\in\mathbb{R},\tag{2.3}$$

where Γ is Gamma function. In the following, we identify "$a.e.\ \omega\in\Omega$" and "$\omega\in\Omega$".

Let

$$v(t,\tau,\omega)=u_t(t,\tau,\omega)+\varepsilon u(t,\tau,\omega)-z(\theta_t\omega)h(x),\quad t\ge\tau,\quad \tau\in\mathbb{R},\tag{2.4}$$

where ε is chosen as

$$\varepsilon=\frac{\alpha\lambda_1+1}{4+2(\alpha\lambda_1+1)\alpha+\frac{1}{\lambda_1}}.$$