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Interaction solutions of a (2+1)-dimensional dispersive long wave system

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ABSTRACT

With the help of the consistent tanh expansion, this paper obtains the interaction solutions between solitons and potential Burgers waves of a (2+1)-dimensional dispersive long wave system. Based on some known solutions of the potential Burgers equation, the multiple resonant soliton wave solutions, soliton–error function wave solutions, soliton–rational function wave solutions and soliton–periodic wave solutions are obtained directly.

1. Introduction

The (2+1)-dimensional dispersive long wave (DLW) system is of the form

$$u_{ty} + v_{xx} + \frac{1}{2}(u^2)_{xy} = 0, \tag{1.1a}$$
$$v_{tx} + u_{xx} + (uv)_{xx} + u_{xxxy} = 0, \tag{1.1b}$$

which is first obtained in Ref. [1] as a compatibility condition for a 'weak' Lax pair. Some integrable properties and exact solutions of the system (1.1) have been studied by several authors [2–10]. For instance, in Ref. [2], system (1.1) is shown to be infinite-dimensional and to have a Kac–Moody–Virasoro structure. Nine types of two-dimensional PDE reductions and 13 types of ODE reductions are given in Ref. [3]. System (1.1) is Lax or inverse scattering transformation integrable [1], however, it does not pass the Painlevé test [5]. Localized excitations, soliton-like solutions, double-like periodic solutions, similarity solutions are studied by variable separation approach, generalized direct method, extended homogeneous balance method, et al. [6–10]. In Ref. [4], the (2+1)-dimensional Broer–Kaup equation

$$q_t = q_{xx} + 2(qp)_x, \tag{1.2a}$$
$$p_{ty} = (p^2)_{xy} + 2q_{xx} - p_{xxy}, \tag{1.2b}$$

is proved to be equivalent to the DLW system (1.1) by transformation

$$v = 4q - 1 - 2p_y, u = -2p. \tag{1.3}$$

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In Refs. [11,12], (1.2) and (1.1) are verified to be consistent Riccati expansion solvable, and their corresponding nonlocal symmetries are also obtained. The aim of this paper is to find the interaction solutions between solitons and potential Burgers waves of system (1.1).

The outline of this paper is as follows: in Section 2, the consistent tanh expansion (CTE) solutions of system (1.1) are presented by means of the CTE method. In Section 3, Schwartz equation (2.3) is decomposed into a pair of equations, and one of which just can be regarded as a variable coefficient potential Burgers equation. Then the multiple resonant soliton wave solutions, soliton–error function wave solutions, soliton–rational function wave solutions and soliton–periodic wave solutions of system (1.1) are obtained directly. Some conclusions are given in Section 4.

2. Consistent tanh expansion solutions

Based on the CTE method [13,14], the CTE solutions of system (1.1) may be

$$u = u_0 + u_1 \tanh(w), \tag{2.1a}$$

$$v = v_0 + v_1 \tanh(w) + v_2 \tanh^2(w), \tag{2.1b}$$

where u_0, u_1, v_0, v_1, v_2 and w are all functions with respect to (x, y, t). By vanishing all the coefficients of the like powers of $\tanh(w)$ after substituting ansatz solutions (2.1) into system (1.1), we obtain

$$u_0 = -\frac{w_t + w_{xx}}{w_x}, u_1 = 2w_x, v_0 = 2w_x w_y - \frac{w_{ty} + w_{xxy}}{w_x} + \frac{(w_t + w_{xx})w_{xy}}{w_x^2} - 1, v_1 = 2w_{xy}, v_2 = -2w_x w_y, \tag{2.2}$$

and the function w only needs to satisfy

$$C_t + 2C_{xx} + S_x - CC_x - 4w_x w_{xx} = 0, \tag{2.3}$$

where the notations C and S are defined as

$$C = \frac{w_t}{w_x}, \; S = \frac{w_{xxx}}{w_x} - \frac{3}{2}\frac{w_{xx}^2}{w_x^2}. \tag{2.4}$$

From (2.1) to (2.4), the system (1.1) has the following CTE solutions

$$u = -\frac{w_t + w_{xx}}{w_x} + 2w_x \tanh(w), \tag{2.5a}$$

$$v = 2w_x w_y - \frac{w_{ty} + w_{xxy}}{w_x} + \frac{(w_t + w_{xx})w_{xy}}{w_x^2} - 1 + 2w_{xy} \tanh(w) - 2w_x w_y \tanh^2(w), \tag{2.5b}$$

with function w satisfies Eq. (2.3). CTE solutions (2.5) contain various interaction solutions among different nonlinear excitations, however, the key of which is to find the solutions of Eq. (2.3).

3. Interaction solutions

It is interesting that Eq. (2.3) can be decomposed into a pair of equations as follows

$$w_t + w_{xx} + 2w_x^2 - cf w_x = 0, \tag{3.1a}$$

$$f_{yt} + f_{xxy} - c(f_x f_y + f f_{xy}) = 0. \tag{3.1b}$$

Eq. (3.1a) can be regarded as a variable coefficient potential Burgers equation, therefore, once the solutions of (3.1a) are known, the interaction solutions among different types of nonlinear waves can be obtained.

It is clear that system (3.1) has the trivial solution

$$w = \kappa x + \iota y + \omega t, \; f = \frac{2\kappa^2 + \omega}{c\kappa}, \tag{3.2}$$

which corresponds to the one soliton solution of system (1.1)

$$\begin{aligned} u &= 2\kappa \tanh(\kappa x + \iota y + \omega t) - \frac{\omega}{\kappa}, \\ v &= -2\kappa\iota \tanh^2(\kappa x + \iota y + \omega t) + 2\kappa\iota - 1. \end{aligned} \tag{3.3}$$

In order to find the interaction solutions between solitons and other waves, we consider the solution of Eq. (3.1a) in the form

$$w = \kappa x + \iota y + \omega t + \varphi, \; \text{with } \varphi \equiv \varphi(x, y, t). \tag{3.4}$$

Under the transformation (3.4) and constant solution $f = \frac{2\kappa^2+\omega}{c\kappa}$ of Eq. (3.1b), Eq. (3.1a) can be rewritten as

$$\varphi_t + \varphi_{xx} + 2\varphi_x^2 + \frac{2\kappa^2 - \omega}{\kappa}\varphi_x = 0. \tag{3.5}$$