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An ISPH scheme for numerical simulation of multiphase flows with complex interfaces and high density ratios

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HIGHLIGHTS

- A simple and robust multiphase scheme is proposed based on the ISPH approach.
- A new interface treatment method is proposed in ISPH using a repulsive force.
- A variety of multiphase problems with single and merging interfaces are considered.
- The scheme is verified against both analytical and numerical results.

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ABSTRACT

Multiphase problems with high density ratios and complex interfaces deal with numerical instabilities and require accurate considerations for capturing the multiphase interfaces. An Incompressible Smoothed Particle Hydrodynamics (ISPH) scheme is presented to simulate such problems. In order to keep the present scheme simple and stable, well-established formulations are used for discretizing the spatial derivatives and a repulsive force is applied at the multiphase interface between particles of different fluids to maintain the interface sharpness. Special considerations are included to overcome the difficulties to model severe physical discontinuities at the interface and surface tension effects are taken into account. Different particle shifting schemes are also tested for a range of problems. Several two phase flows are investigated and the presented scheme is validated against both analytical and numerical solutions. A detailed study is also carried out on the influence of the repulsive force in an ISPH scheme showing that this simple treatment efficiently enhances the interface capturing features. The comparisons indicate that the proposed scheme is robust and capable of simulating a wide range of multiphase problems with complex interfaces including low to high ratios for density and viscosity.

1. Introduction

Multiphase phenomena, where two or more fluids are present simultaneously in a domain are ubiquitous in many natural and industrial problems. Different fluids are separated by an interface and the physical parameters are discontinuous across the interface. The physical discontinuities, the evolution of the interface and the mutual influence of different phases at the interface introduce the study of these problems as one of the most challenging topics in computational fluid dynamics

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(CFD). Most of the numerical investigations of multiphase flows are based on Eulerian mesh-based methods where, because of the mesh constraints, they need specific interface treatment algorithms to obtain valid numerical results. Volume of Fluid (VOF) [1], Level-Set (LS) [2] and Front Tracking (FT) [3] are well-known methods for interface capturing in mesh-based simulations of multiphase flows.

In recent years, Lagrangian mesh-free methods have emerged as an attractive alternative for mesh-based methods in simulation of multiphase flows. Smoothed particle hydrodynamics (SPH) is a more advanced member of this category, which has been proposed in late 1970s for astrophysical applications. SPH is a fully Lagrangian particle-based method which, owing to its Lagrangian nature, tracks the multiphase interfaces without any need for an additional algorithm. This advantage nominates SPH as an appropriate candidate for simulating multiphase flows and has attracted a great deal of attention in recent decade [4–8]. A comprehensive review on the interfacial flow simulations using SPH can be found in [9].

In order to simulate an incompressible multiphase flow, there are two general approaches to impose the incompressibility constraint in SPH. The most widely used approach is the weakly compressible SPH (WCSPH), where pressure is related to density using an equation of state allowing a small compressibility. The other approach is the incompressible SPH (ISPH), based on the projection method for numerical solution of the Navier–Stokes equations [10], where pressure is calculated by solving the elliptic pressure Poisson equation (PPE) at each time step enforcing the incompressibility. Two-phase compressible–incompressible schemes are the recent alternative approaches (see e.g. [8,11]), where the gaseous and the liquid phases are considered as a compressible fluid and an incompressible fluid, respectively. Indeed, the treatment of the gaseous phase depends on the ratio of the pressure forces in this phase to the inertia of the liquid phase [12]. If this ratio is large, the influence of the gaseous phase is considerable and can be treated as an incompressible medium. On the contrary, in small ratios the gaseous phase effects are negligible and a single-phase model can be adopted. In the intermediate cases, the gaseous phase has to be modeled as a compressible fluid.

Although it is well-known that the ISPH approach leads to noise-free predictions for the pressure field [13,14] and accurately simulates complex problems (see e.g. [15–17]), it has rarely been used to simulate multiphase flows (see e.g. [18–22]). Except for the coupled ISPH-WCSPH model in [8], among the previous ISPH studies on multiphase simulations, only the work by Zainali et al. [20] reported the simulations with high density ratios up to 1:1000 and viscosity ratios up to 1:100. Therefore, further studies are necessary on ISPH simulation of multiphase problems characterized by high density ratios.

The present work proposes an efficient ISPH algorithm based on the velocity divergence-free form of the projection method to simulate multiphase flows with complex interfaces and high density ratios. Well-established SPH discretized formulations are used for first and second order derivatives, and interface sharpness is simply maintained using a repulsive force applied between particles of different fluids. The spurious fragmentations of interface are prevented by this simple treatment and there is no need for the corrective tensor, proposed by Zainali et al. [20], to remove particle inconsistencies. The proposed tensor needs an inversion, which might lead to singularities. Continuum surface force (CSF) [23] model is employed to take the surface tension effects into account and special smoothing methods are used to circumvent the physical discontinuities at the multiphase interface. Furthermore, different particle shifting schemes are tested alongside different kernel functions for a range of problems with and without the presence of free-surface. Several test cases have been considered to validate the proposed algorithm against both analytical and numerical results. Furthermore, because of the importance of the repulsive force, its influence on captured interfaces is investigated in more detail.

The manuscript is organized as follows: Section 2 describes the mathematical details of the proposed multiphase ISPH scheme. The numerical results are then presented and discussed in Section 3. Finally, the concluding remarks of the present study are highlighted in Section 4.

2. Mathematical model

2.1. Governing equations

For an incompressible flow, the mass and momentum conservation equations can be written respectively as:

$$\nabla \cdot \mathbf{u} = 0, \tag{1}$$

$$\frac{\mathrm{D}\mathbf{u}}{\mathrm{D}t} = -\frac{1}{\rho}\nabla p + \nu \nabla^2 \mathbf{u} + \mathbf{g} + \frac{1}{\rho}\mathbf{F}_s, \tag{2}$$

where D is the material (Lagrangian) derivative, ρ is the density, t is the time, $\mathbf{u}$ is the velocity field, p is the pressure, ν is the kinematic viscosity, $\mathbf{g}$ is the gravitational acceleration and $\mathbf{F}_s$ is the volumetric surface tension force.

2.2. SPH basics

SPH is a fully Lagrangian method, which describes the fluid flow by a finite number of particles. The particles carry mass and other physical parameters following the flow field and are considered as interpolation points. The main idea of SPH is