ARTICLE IN PRESS

Computers and Mathematics with Applications ▮ (▮▮▮▮) ▮▮▮–▮▮▮

Contents lists available at ScienceDirect

Computers and Mathematics with Applications

journal homepage: www.elsevier.com/locate/camwa

Nonexistence of global solutions for a class of nonlocal in time and space nonlinear evolution equations

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ARTICLE INFO

Article history:
Received 16 August 2017
Received in revised form 30 November 2017
Accepted 4 January 2018
Available online xxxx

Keywords:
Nonexistence
Global solution
Caputo fractional derivative

ABSTRACT

In this paper, we study the nonlocal nonlinear evolution equation

$$ {}^C D^{\alpha}_{0|t} u(t,x) - (J * |u| - |u|)(t,x) + {}^C D^{\beta}_{0|t} u(t,x) = |u(t,x)|^p, \quad t>0,\ x\in\mathbb{R}^d, $$

where $1<\alpha<2$, $0<\beta<1$, $p>1$, $J:\mathbb{R}^d\to\mathbb{R}_+$, $*$ is the convolution product in $\mathbb{R}^d$, and ${}^C D^{q}_{0|t}$, $q\in\{\alpha,\beta\}$, is the Caputo left-sided fractional derivative of order q with respect to the time t. We prove that the problem admits no global weak solution other than the trivial one with suitable initial data when $1<p<1+\frac{2\beta}{d\beta+2(1-\beta)}$. Next, we deal with the system

$$ \begin{cases} {}^C D^{\alpha}_{0|t} u(t,x) - (J * |u| - |u|)(t,x) + {}^C D^{\beta}_{0|t} u(t,x) = |v(t,x)|^p, & t>0,\ x\in\mathbb{R}^d,\\ {}^C D^{\alpha}_{0|t} v(t,x) - (J * |v| - |v|)(t,x) + {}^C D^{\beta}_{0|t} v(t,x) = |u(t,x)|^q, & t>0,\ x\in\mathbb{R}^d, \end{cases} $$

where $1<\alpha<2$, $0<\beta<1$, $p>1$, and $q>1$. We prove that the system admits non global weak solution other than the trivial one with suitable initial data when $1<pq<1+\frac{2\beta}{d\beta+2(1-\beta)}\max\{p+1,q+1\}$. Our approach is based on the test function method.

1. Introduction

Nonlinear evolution equations involving a convolution product have been studied by many mathematicians (see, for example [1–12]).

In [6], García-Melián and Quirós considered the nonlocal diffusion problem

$$ \begin{cases} u_t(t,x) = (J*u-u)(t,x) + u^p(t,x), & t>0,\ x\in\mathbb{R}^d,\\ u(0,x)=u_0(x), & x\in\mathbb{R}^d, \end{cases} \tag{1} $$

where $J:\mathbb{R}^d\to\mathbb{R}_+$ is a compactly supported nonnegative function with unit integral, $p>1$, $u_0\in L^1(\mathbb{R}^d;\mathbb{R}_+)\cap L^\infty(\mathbb{R}^d;\mathbb{R}_+)$, and $*$ is the standard convolution product in $\mathbb{R}^d$. They proved that Problem (1) has as critical exponent

$$ p_c = 1+\frac{2}{d}, $$

which is the Fujita exponent [13] for the classical nonlinear heat equation $u_t=\Delta u+u^p$.

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https://doi.org/10.1016/j.camwa.2018.01.001
0898-1221/

Yang [12] considered the nonlinear coupled nonlocal diffusion system

$$\begin{cases} u_t(t,x) = (J*u-u)(t,x) + v^p(t,x), & t>0,\ x\in\mathbb{R}^d,\\ v_t(t,x) = (J*v-v)(t,x) + u^q(t,x), & t>0,\ x\in\mathbb{R}^d,\\ (u(0,x),v(0,x)) = (u_0(x),v_0(x)), & x\in\mathbb{R}^d, \end{cases} \tag{2}$$

where $p>1$, $q>1$, and $(u_0,v_0)\in L^\infty(\mathbb{R}^d;\mathbb{R}_+)\times L^\infty(\mathbb{R}^d;\mathbb{R}_+)$. He established that the critical Fujita curve is given by

$$(pq)^* = 1 + \frac{2}{d}\max\{p+1,q+1\},$$

which is also the Fujita curve for the coupled heat system $u_t = \Delta u + v^p$ and $v_t = \Delta v + u^q$, obtained by Escobedo and Herrero [14].

In [10], we studied the following fractional evolution equation with nonlocal diffusion

$$\begin{cases} D^\alpha_{0|t}(u(t,x)-u_0) = (J*u-u)(t,x) + u^p(t,x), & t>0,\ x\in\mathbb{R}^d,\\ u(0,x) = u_0(x), & x\in\mathbb{R}^d, \end{cases} \tag{3}$$

where $0<\alpha<1$, $D^\alpha_{0|t}$ is the Riemann–Liouville fractional derivative of order α with respect to the time t, $p>1$, and $u_0\in L^1_{loc}(\mathbb{R}^d;\mathbb{R}_+)$. Under certain assumptions on the function $J:\mathbb{R}^d\to\mathbb{R}_+$, we proved that if

$$1<p<1+\frac{2\alpha}{d\alpha+2(1-\alpha)},$$

then Problem (3) admits non global weak solution other than the trivial one. In the same reference, we studied also the system

$$\begin{cases} D^\alpha_{0|t}(u(t,x)-u_0) = (J*u-u)(t,x) + v^p(t,x), & t>0,\ x\in\mathbb{R}^d,\\ D^\alpha_{0|t}(v(t,x)-v_0) = (J*v-v)(t,x) + u^q(t,x), & t>0,\ x\in\mathbb{R}^d,\\ (u(0,x),v(0,x)) = (u_0(x),v_0(x)), & x\in\mathbb{R}^d, \end{cases} \tag{4}$$

where $0<\alpha<1$, $p>1$, $q>1$, and $(u_0,v_0)\in L^1_{loc}(\mathbb{R}^d;\mathbb{R}_+)\times L^1_{loc}(\mathbb{R}^d;\mathbb{R}_+)$. We proved that if

$$1<pq<1+\frac{2\alpha}{d\alpha+2(1-\alpha)}\max\{p+1,q+1\},$$

then Problem (4) admits non global weak solution other than the trivial one.

Motivated by the above cited works, in this paper we are first concerned with the following nonlocal in time and space nonlinear evolution equation

$$\begin{cases} {}^CD^\alpha_{0|t}u(t,x) - (J*|u|-|u|)(t,x) + {}^CD^\beta_{0|t}u(t,x) = |u(t,x)|^p, & t>0,\ x\in\mathbb{R}^d,\\ (u(0,x),u_t(0,x)) = (u_0(x),u_1(x)), & x\in\mathbb{R}^d, \end{cases} \tag{5}$$

where $1<\alpha<2$, $0<\beta<1$, $p>1$, $J:\mathbb{R}^d\to\mathbb{R}_+$, $*$ is the convolution product in $\mathbb{R}^d$, ${}^CD^q_{0|t}$, $q\in\{\alpha,\beta\}$, is the Caputo left-sided fractional derivative of order q with respect to the time t, and $u_i\in L^1(\mathbb{R}^d)$, $i=0,1$. Problem (5) can be viewed either as a time fractional heat equation with a fractional time relaxation term [15] or a time fractional wave equation with a fractional damping [16–19] or a nonlinear fractional telegraph equation [20,21]. Using the test function method developed by Mitidieri and Pohozaev [22,23] (see also [24–27]), under suitable initial data and certain conditions imposed on the function J, we shall prove that Problem (5) admits non global weak solution other than the trivial one when

$$1<p<1+\frac{2\beta}{d\beta+2(1-\beta)}.$$

Observe that the same result was obtained in [10] for Problem (3) (with $\alpha=\beta$). The same phenomena occurs for the nonlinear wave equation with damping $u_{tt}-\Delta u+u_t=|u|^p$, which admits as critical exponent $1+\frac{2}{d}$ (see [28,29]), that is, the Fujita critical exponent for the nonlinear heat equation $u_t-\Delta u=|u|^p$. For the local existence (in time) for Problem (5), it can be proved using standard arguments (Banach fixed point theorem), see for example [30]. The next part of the paper is devoted to study the system

$$\begin{cases} {}^CD^\alpha_{0|t}u(t,x) - (J*|u|-|u|)(t,x) + {}^CD^\beta_{0|t}u(t,x) = |v(t,x)|^p, & t>0,\ x\in\mathbb{R}^d,\\ {}^CD^\alpha_{0|t}v(t,x) - (J*|v|-|v|)(t,x) + {}^CD^\beta_{0|t}v(t,x) = |u(t,x)|^q, & t>0,\ x\in\mathbb{R}^d,\\ (u(0,x),u_t(0,x)) = (u_0(x),u_1(x)),\ (v(0,x),v_t(0,x)) = (v_0(x),v_1(x)), & x\in\mathbb{R}^d, \end{cases} \tag{6}$$

where $1<\alpha<2$, $0<\beta<1$, $p>1$, $q>1$, and $(u_i,v_i)\in L^1(\mathbb{R}^d)\times L^1(\mathbb{R}^d)$, $i=0,1$. Under suitable initial data and certain conditions imposed on the function J, we shall prove that Problem (6) admits non global weak solution other than the trivial