Computers and Mathematics with Applications ▮ (▮▮▮▮) ▮▮▮–▮▮▮

Contents lists available at ScienceDirect

Computers and Mathematics with Applications

journal homepage: www.elsevier.com/locate/camwa

A discrete Boltzmann equation model for two-phase shallow granular flows

Michele La Rocca [a,*], Andrea Montessori [a], Pietro Prestininzi [a], Lakshmanan Elango [b]

[a] Department of Engineering, Roma Tre University, Via Vito Volterra 62, 00146 Rome, Italy
[b] Department of Geology, Anna University, Chennai, India

HIGHLIGHTS

- A novel multispeed discrete Boltzmann model is developed for two-phase granular flows.
- A physically based formulation for the interphase drag force is considered.
- A thorough validation of the model is performed.

ARTICLE INFO

Article history:
Received 29 March 2017
Received in revised form 28 December 2017
Accepted 6 January 2018
Available online xxxx

Keywords:
Discrete Boltzmann equation
Granular flows
Shallow water model
Interphase drag

ABSTRACT

In this paper a Discrete Boltzmann Equation model (hereinafter DBE) is proposed as solution method of the two-phase shallow granular flow equations, a complex nonlinear partial differential system, resulting from the depth-averaging procedure of mass and momentum equations of granular flows. The latter, as e.g. a debris flow, are flows of mixtures of solid particles dispersed in an ambient fluid.

The reason to use a DBE, instead of a more conventional numerical model (e.g. based on Riemann solvers), is that the DBE is a set of linear advection equations, which replaces the original complex nonlinear partial differential system, while preserving the features of its solutions. The interphase drag function, an essential characteristic of any two-phase model, is accounted for easily in the DBE by adding a physically based term. In order to show the validity of the proposed approach, the following relevant benchmark tests have been considered: the 1D simple Riemann problem, the dam break problem with the wet–dry transition of the liquid phase, the dry bed generation and the perturbation of a state at rest in 2D. Results are satisfactory and show how the DBE is able to reproduce the dynamics of the two-phase shallow granular flow.

1. Introduction

A granular flow can be defined as a dense fluid–particle flow in which the direct particle–particle interactions are a dominant feature, characterizing the flow patterns. A typical example of granular flow of geophysical interest is the debris flows, ultra-high concentrated suspensions of sediments in water. Snow avalanches and landslides are further examples of granular flows. A deeper understanding of the dynamics of granular flows is pivotal so as to mitigate hazards and prevent

* Corresponding author.
E-mail address: michele.larocca@uniroma3.it (M. La Rocca).

https://doi.org/10.1016/j.camwa.2018.01.010

the dramatic consequences these natural phenomena entail. This motivates the considerable modeling effort done at various levels in the past decades. A recent review of the topic is given in [1].

A simplified but technically important mathematical representation of granular flows can be obtained by applying the shallow water theory, i.e. by depth-averaging the equations of mass and momentum balance. The validity of the extension of the shallow water theory to granular flows lies in the fact that in these flows a spatial dimension, the flow depth, is usually much smaller than the others. In the seminal work of Pitman and Le [2] the depth-averaged approach is obtained starting from the two-phase Navier–Stokes equations for the mixture of Coulomb material and Newtonian fluid. The model of Pitman and Le is the base of the two-phase shallow granular flow models used in successive works, as e.g. [3,4].

The two-phase shallow granular flow equations developed by Pitman and Le [2] and successively adopted in [4], where basal and bottom friction are neglected, form a complex system of six partial nonlinear differential equations. Among forcing terms, the interphase drag function is an essential feature, necessary for a correct coupling of the two phases and whose definition has to be physically consistent.

In literature, the most common numerical approach to the two-phase shallow granular flow equations is based on Riemann solver or Godunov methods, as in [3,4], able to deal with discontinuous solutions, as e.g. propagating shocks. The interphase drag is often overlooked and introduced as a mere numerical stabilization term.

Aiming at obtaining a more agile and computational efficient method, in this paper we propose a Discrete Boltzmann Equation model (hereinafter DBE) as an alternative approach for the formulation and the solution of the two-phase shallow granular flow model. The DBE belongs to the family of mesoscopic approaches, as e.g. the Lattice Boltzmann, recently developed and successfully applied to several flow problems, as the consideration of fixed and moving immersed boundaries [5–7], of three-dimensional multiphase flows [8] and of flows in porous media [9]. The success of these approaches is due to the fact that, deriving from the Boltzmann kinetic equation, the convective term is linear, while nonlinearities are retained only in the collisional term. This feature implies considerable coding facility and computational efficiency [10]. By means of the Chapman–Enskog multiscale perturbation procedure, it is possible to show that the results obtained by the considered mesoscopic approach are equivalent to those obtained by the conventional macroscopic approach (the two-phase shallow granular flow equations in the present case). The reader is addressed to [11,12] for an introduction to mesoscopic (mainly Lattice Boltzmann) approaches and to [10] for an exhaustive review on their application.

The extension of the mesoscopic models to multi-layered shallow water flows has been done originally by Tubbs et al. [13], who extended the Lattice Boltzmann shallow water theory of Zhou [14]. La Rocca et al. [15] applied the theory of Tubbs to two-layered shallow water flows, consisting of two shallow layers of immiscible liquid with different densities. However the Lattice Boltzmann shallow water theory has a main shortcoming: it is not able to handle supercritical flows, i.e. when the Froude number of the flow approaches and becomes greater than one. When this condition is met, the equivalence between the Lattice Boltzmann and the conventional shallow water theory fails, as the difference between the two formulations increases with the Froude number. Within the field of kinetic-based methods, a possible solution is to resort to full "continuous" Boltzmann methods [16], but care should be taken not to spoil the advantages connected to the phase space discretization. This shortcoming can also be overcome by adopting a more complex discretization of the Boltzmann's equation as shown in [17], where the proposed DBE formulation is able to simulate accurately single-layer trans- and super-critical flows, both one and two dimensional. The difference between the DBE and the Lattice Boltzmann Equation however, is that the former does not admit the lattice structure for space–time discretization, then needing the adoption of traditional numerical discretization methods, as e.g. the finite difference method.

The DBE developed in [17] is here extended and applied to the formulation and the solution of the two-phase shallow granular flow model. The main advantage with respect to more traditional methods is the extreme simplicity of the numerical treatment of the model.

As for the interphase drag, the proposed DBE is provided with a physically consistent model, developed starting from the work of Gidaspow [18] and accounting for the interaction between solid particles and the liquid phase [19].

To the authors best knowledge, such formulation of the interphase drag is alternative to those usually adopted in literature and considered as mere numerical remedies, necessary to ensure stability [4].

The aim of this work is then to show the validity of the DBE in modeling the two-phase shallow granular flow and to investigate the effect of the interphase drag force term, based on the modified model of Gidaspow.

The paper is structured as follows: firstly, the mathematical model for the two-phase shallow granular flow and the DBE formulation is presented; secondly, the interphase drag model is presented in detail; thirdly, relevant benchmark cases are described and results are presented and discussed; fourthly, conclusions are drawn.

2. The two-phase shallow granular flow model and the discrete Boltzmann equation model

The two-phase shallow granular flow model considered here is developed in [20] and [4] starting from the depth average model of Pitman and Le [2]. The model describes the dynamics of a shallow layer of mixture, height h, made of solid particles, average diameter D, density ρ_s, immersed in an ambient liquid, density ρ_l.

As in [4], let ϕ denote the volume fraction and h the depth of the shallow granular flow. Thus the heights of the liquid and the solid phases h_l, h_s can be expressed as:

$$\begin{aligned} h_l &= (1-\phi)h \\ h_s &= \phi h. \end{aligned} \tag{1}$$