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The computation of strain rate tensor in multiple-relaxation-time lattice Boltzmann model

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ABSTRACT

The multiple-relaxation-time (MRT) lattice Boltzmann (LB) model is an important class of LB model with lots of advantages over the traditional single-relaxation-time (SRT) LB model. Generally, the computation of strain rate tensor is crucial for the MRT-LB simulations of some complex flows. At present, only two formulae are available to compute the strain rate tensor in the MRT LB model. One is to compute the strain rate tensor using the non-equilibrium parts of macroscopic moments (Yu formula). The other is to compute the strain rate tensor using the non-equilibrium parts of density distribution functions (Chai formula). The mathematical expressions of these two formulae are so different that we do not know which formula to choose for computing the strain rate tensor in the MRT LB model. To overcome this problem, this paper presents a theoretical study of the relationship between Chai and Yu formulae. The results show that the Yu formula can be deduced from the Chai formula, although they have their own advantages and disadvantages. In particular, the Yu formula is computationally more efficient, while the Chai formula is applicable to more lattice patterns of the MRT LB models. Furthermore, the derivation of the Yu formula in a particular lattice pattern from the Chai formula is more convenient than that proposed by Yu et al.

1. Introduction

The lattice Boltzmann equation (LBE), as a mesoscopic numerical method, has been widely used to simulate various complex fluid flows and has gained significant success because of its distinct advantages such as the natural parallelism of algorithm, simplicity of programming and ease of dealing with complex boundary conditions [1–3].

In recent years, the LBE method has been successfully used to study, for example, micro and nano flows [4,5], blood flows [6–9], non-Newtonian fluid flows [10–17], multiphase flows [18,19], as well as turbulent flows based on either direct numerical simulation or large eddy simulation [20–25]. This success has demonstrated the wide applicability of the LBE method. More importantly, in most of the above studies, the computation of strain rate tensor is of key importance for obtaining reliable predictions, and it has hence received increasing attention in the LB modelling over the past decade [13,14,23,26–29].

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For the SRT LB model, it computes the strain rate tensor by using the non-equilibrium parts of density distribution functions [6,8,18,26,30]. This particular method is local [30] and has second-order accuracy in space. Thus, it is very suitable for the studies of flows in complex geometries and has a great advantage over the traditional second-order finite difference method. Although the computation of the strain rate tensor has been extensively studied for the SRT LB (also called LBGK) model, it has been investigated for the MRT LB model, only to a small extent.

Yu et al. proposed to compute the strain rate tensor with the non-equilibrium parts of macroscopic moments (Yu formula) for the large eddy simulation (LES) of turbulent jets by using the MRT LB model with nineteen discrete velocities in three dimensions (D3Q19) [23]. Based on the Yu formula, Premnath et al. developed the formula to compute the strain rate tensor with the external force effect in the LES of turbulent flows [31]. Pattison et al. used the formula of Premnath et al. to compute the strain rate tensor in their LES of turbulent flow through a straight square duct as driven by a pressure gradient [32]. Also, some other efforts were made to use the formulae developed by Yu et al. and Premnath et al. to compute the strain rate tensor in the LES of turbulent flows [33,34].

More recently, Chai et al. proposed the other formula to compute the strain rate tensor with the non-equilibrium parts of density distribution functions (Chai formula) for the simulation of non-Newtonian fluid flows, where the MRT LB model with nine discrete velocities in two dimensions (D2Q9) [14] was adopted. After that work, Chai formula was widely used to compute the strain rate tensor in the MRT-LB simulation of non-Newtonian fluid flows [35–37].

To the best knowledge of the authors, the Yu formula and Chai formula are the only two formulae currently available to compute the strain rate tensor in the MRT LB model. The forms of two formulae are so different that we are not sure which formula to choose in our LB simulations. To overcome this problem, we first derive these two formulae under the convective scaling and diffusive scaling [38]. Then, we study the relationship of these two formulae and their relative advantages and disadvantages. The spatial accuracy of Yu and Chai formulae is also compared by simulating the two-dimensional Taylor–Green vortex flow. Based on these efforts, we give our recommendations on which formula to choose for computing the strain rate tensor in the MRT LB model.

The paper is organized as follows. We first present the He–Luo MRT LB model with D3Q19 lattice pattern [39] as the starting point. Second, based on either the Chapman–Enskog analysis or the asymptotic analysis [28], we then deduce the Yu and Chai formulae, which are used for computing the strain rate tensor in the D3Q19 MRT LB model. Third, we study the equivalence and difference of these two formulae. The suggestions on the choice of the formulae are also given for computing the strain rate tensor in the MRT LB model. Finally, conclusions are made.

2. He–Luo D3Q19 MRT LB model

In this paper, we compare the two currently available formulae (i.e. the Yu formula and the Chai formula) which are for the computation of the strain rate tensor in the MRT LB model. The macroscopic equilibrium moments of the MRT LB model are chosen to be derived from the equilibrium distribution functions of the SRT LB (also called LBGK) model as proposed by He and Luo [40], which is referred to as the He–Luo MRT LB model. In the following, we take the He–Luo MRT LB model with D3Q19 lattice as an example to carry out the comparison.

The evolution equation of He–Luo D3Q19 MRT LB model is

$$f_\alpha(\boldsymbol{x}+\boldsymbol{c}_\alpha\delta_t, t+\delta_t) - f_\alpha(\boldsymbol{x}, t) = -\sum_{i=0}^{18} \Lambda_{\alpha i}(f_i(\boldsymbol{x}, t) - f_i^{(eq)}(\boldsymbol{x}, t)), \ \ \alpha = 0-18, \tag{1}$$

where $f_l(\boldsymbol{x}, t)$ and $f_l^{(eq)}(\boldsymbol{x}, t)$ $(l = \alpha, i)$ are the distribution function and equilibrium distribution function of particles with the velocity $\boldsymbol{c}_l$ at the node $\boldsymbol{x}$ and the time t, $\Lambda_{\alpha i}$ is the element located in the αth row and ith column of 19×19 collision matrix $\boldsymbol{\Lambda}$. For the He–Luo MRT model, the equilibrium distribution function is chosen as

$$f_i^{(eq)}(\boldsymbol{x}, t) = \omega_i \left[\rho + \rho_0 \left(\frac{\boldsymbol{c}_i\cdot\boldsymbol{u}}{c_s^2} + \frac{(\boldsymbol{c}_i\cdot\boldsymbol{u})^2}{2c_s^4} - \frac{|\boldsymbol{u}|^2}{2c_s^2}\right)\right], \tag{2}$$

where

$$\omega_i = \begin{cases} 1/3, & i = 0, \\ 1/18, & i = 1-6, \\ 1/36, & i = 7-18, \end{cases} \tag{3}$$

$\boldsymbol{c}_i$ is defined as

$$\{\boldsymbol{c}_0, \boldsymbol{c}_1, \ldots, \boldsymbol{c}_{18}\} = \left\{\begin{matrix} 0 & 1 & -1 & 0 & 0 & 0 & 0 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 & 0 & 1 & 1 & -1 & -1 & 0 & 0 & 0 & 0 & 1 & -1 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 \end{matrix}\right\} c, \tag{4}$$