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A finite difference/finite element technique with error estimate for space fractional tempered diffusion-wave equation

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ABSTRACT

An efficient numerical technique is proposed to solve one- and two-dimensional space fractional tempered fractional diffusion-wave equations. The space fractional is based on the Riemann–Liouville fractional derivative. At first, the temporal direction is discretized using a second-order accurate difference scheme. Then a classic Galerkin finite element is employed to obtain a full-discrete scheme. Furthermore, for the time-discrete and the full-discrete schemes error estimate has been presented to show the unconditional stability and convergence of the developed numerical method. Finally, two test problems have been illustrated to verify the efficiency and simplicity of the proposed technique.

1. Introduction

The fractional PDEs play important and basic role in modeling and simulating of natural and practical phenomena [1]. In the meantime, the fractional equations based on the space fractional type are important and have more difficulty for simulating and analysis.

Authors of [2] applied the finite element scheme and interpolating element free Galerkin technique for solving the 2D time fractional diffusion-wave equation on the irregular domains. A new combination of alternating direction implicit (ADI) method with the Galerkin finite element technique is proposed in [3] to solve 2D fractional diffusion-wave equation. The main aim of [4] is to develop a numerical procedure based on the mixed finite element method for the time-fractional fourth-order partial differential equation (PDE). Authors of [5] developed a two-grid algorithm based on the mixed finite element (MFE) method for a nonlinear fourth-order reaction–diffusion equation with the time-fractional derivative of Caputo-type. A new Crank–Nicolson alternating direction implicit (ADI) Galerkin finite element method for the 2D fractional FitzHugh–Nagumo monodomain model is developed in [6]. Authors of [7] proposed a new numerical scheme for solving the time–space fractional telegraph equation based on the Galerkin finite element method for the spatial Riemann–Liouville fractional derivative and finite difference schemes for the temporal Caputo derivatives.

The main aim of [8] is to investigate a new numerical procedure for solving a class of two-dimensional Riesz space fractional diffusion equations based on the Galerkin finite element method and backward difference method with error

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analysis. Authors of [9] solved two types of space–time fractional diffusion equations (STFDEs) on a finite domain and then they solved them using the finite element method. In [10] a numerical solution for space–time fractional diffusion equation is considered whereas the spatial discretization is effected using a finite element method and the θ-scheme is used for temporal discretization. An in-depth numerical analysis of spatial fractional advection–diffusion equation (SFADE) utilizing the finite element method (FEM) is presented in [11].

The backward Euler and Crank–Nicolson–Galerkin fully-discrete approximate schemes combined with the finite element method for two-dimensional space-fractional advection–dispersion equations [12]. Two novel numerical methods based on the finite volume and finite element methods with a nonlocal operator (using nodal basis functions) for the space-fractional Boussinesq equation are derived in [13]. Author of [14] developed the finite element method for the numerical resolution of the space and time fractional Fokker–Planck equation.

An error analysis of a finite element method for the space-fractional parabolic equation has been proposed in [15]. Author of [16] solved 2D Riesz space-fractional damped sine-Gordon equation using a finite-difference discretization. Authors of [17,18] developed a high order numerical scheme for multi-dimensional variable-order fractional Schrödinger equations using the new operational matrices based on the shifted Jacobi polynomials. An unconditionally energy stable Fourier spectral scheme is developed in [19] to solve the fractional-in-space Cahn–Hilliard equation with periodic or Neumann boundary conditions.

Authors of [20] proposed the point interpolation method (PIM), a meshfree method, to solve space fractional advection–dispersion equations, where the polynomial point-interpolation functions and their fractional derivatives with explicit expressions are substituted into Galerkin weak form of SFADE to obtain the discrete approximation system. The main aim of [21] is to find a finite difference scheme for space tempered fractional diffusion equations by using the generation function of the matrix and Weyl's theorem . Also the stability and convergence of the derived schemes are strictly proved. The main aim of [22] is to propose some efficient and robust numerical methods to compute the ground states and dynamics of Fractional Schrödinger Equation (FSE) with a rotation term and nonlocal nonlinear interactions. Authors [23] proposed a simple and unconditionally stable time-split Gauss–Seidel projection (GSP) method for the space fractional Landau–Lifshitz (FLL) equations. A fast finite volume method has been developed in [24] for variable-coefficient, conservative space-fractional diffusion equations in convex domains via a volume-penalization approach. A finite element method is developed in [25] to solve 2D-space fractional diffusion equations and FitzHugh–Nagumo model with nonlinear source term.

Authors of [26] proposed a fast and accurate method for numerical solutions based on the Fourier spectral method in space for solving space fractional reaction–diffusion equations. Authors of [27] developed a Petrov–Galerkin (PG) spectral method for Fractional Initial-Value Problems (FIVPs) and they obtained solutions to FIVPs and Fractional Final VPs (FFVPs) in terms of the new fractional (non-polynomial) basis functions, called "Jacobi polyfractonomials". A spectral element method for a time- and space-fractional advection equation is proposed in [28].

A fourth-order finite difference scheme is presented for solving one- and two-dimensional time–space fractional sub-diffusion equations [29]. Authors of [30] developed a second-order finite difference scheme for fractional diffusion equation. A novel compact operator is derived for the approximation of the Riesz derivative for solving fractional Schrödinger equation [31]. Authors of [32] developed a new numerical scheme based on the finite difference scheme in time variable and finite element method in spatial direction for solving two-dimensional space and time fractional Bloch–Torrey equations. A high order finite difference scheme for a two-dimensional fractional Klein–Gordon equation subject to Neumann boundary conditions is proposed in [33] with analysis of stability and convergence.

The main aim of [34] is to develop an unconditionally stable second-order finite difference scheme for solving the space–time tempered fractional diffusion-wave equation. Also, authors of [35] proposed an unconditionally stable fourth-order finite difference procedure to numerical simulation of the space–time tempered fractional diffusion-wave equation.

Authors of [36] developed a fourth-order difference method for the fractional diffusion-wave equation. Author of [37] constructed a new difference analog of the Caputo fractional derivative and they presented stability of the suggested scheme and also its convergence in the grid L_2-norm. Author of [38] proposed a difference scheme for solving the Dirichlet and Robin boundary value problems based on the multi-term variable-distributed order diffusion equation. Authors of [39] employed the hybrid functions approximation for solving the distributed fractional differential equations. Alternative approach is proposed in [1] where the main idea behind it is applying a semi-analytical procedure to find the solution of nonlinear fractional partial differential equations. Author of [40] proposed three fully implicit finite difference schemes, two fully explicit finite difference techniques, an alternating direction implicit procedure and the Barakat and Clark type explicit formula for solving the two-dimensional Schrodinger equation with Dirichlet's boundary conditions. The main aim of [41] is to present a finite element method for solving 2D Rayleigh-Stokes model with fractional derivative on irregular domains such as circular, L-shaped and a unit square with a circular and square hole. Authors of [42] proposed a new operational matrix for solving fractional-order differential equations. A second-order finite difference scheme has been developed in [43] for solving 3D space and time fractional Bloch–Torrey equation. Authors of [44] proposed several difference schemes for both one-dimensional and two-dimensional space and time fractional Bloch–Torrey equations. The main aim of [45] is to develop a fully discrete difference scheme for a diffusion-wave system by introducing two new variables to transform the original equation into a low order system of equations.