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Computers and Mathematics with Applications

journal homepage: www.elsevier.com/locate/camwa

Simplest equation method for some time-fractional partial differential equations with conformable derivative☆

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ARTICLE INFO

Article history:
Received 29 July 2016
Received in revised form 21 June 2017
Accepted 21 January 2018
Available online xxxx

Keywords:
Conformable fractional derivative
Simplest equation method
Fractional differential equations
Traveling wave transformation

ABSTRACT

The conformable fractional derivative was proposed by R. Khalil et al. in 2014, which is natural and obeys the Leibniz rule and chain rule. Based on the properties, a class of time-fractional partial differential equations can be reduced into ODEs using traveling wave transformation. Then the simplest equation method is applied to find exact solutions of some time-fractional partial differential equations. The exact solutions (solitary wave solutions, periodic function solutions, rational function solutions) of time-fractional generalized Burgers equation, time-fractional generalized KdV equation, time-fractional generalized Sharma–Tasso–Olver (FSTO) equation and time-fractional fifth-order KdV equation, $(3 + 1)$-dimensional time-fractional KdV–Zakharov–Kuznetsov (KdV–ZK) equation are constructed. This method presents a wide applicability to solve some nonlinear time-fractional differential equations with conformable derivative.

1. Introduction

In recent years, fractional differential equations (FDEs), which are used to describe several phenomena in many fields of sciences, such as physics, electrochemistry, chemistry, biology, control theory and other areas [1–5], have been widely concerned. To find exact solutions of FDEs is an important aspect of scientific research. Therefore, a lot of scholars have devoted to studying the exact solutions of FDEs and developed some techniques to deal with FDEs, such as Lie symmetry analysis method [6–9], the differential transform method [10], the iteration method [11,12], etc.

There are several definitions for fractional derivative [1–3], such as Riemann–Liouville, Caputo fractional, Grunwald–Letnikov et al. In 2014, R. Khalil et al. [13] presented a new well-behaved definition of fractional derivative called conformable fractional derivative. So many properties related to this new definition were studied [13–15]. For instance, T. Abdeljawad [14] developed the definitions and proved some properties, such as fractional versions of chain rule, Gronwall inequality, integration by parts, fractional power series expansions, Laplace transforms for certain fractional type functions, and so forth. In Ref. [15], T. Abdeljawad et al. used the fractional semigroups approach for studying abstract conformable Cauchy problems. At the end of reference [14], the author puts forward an open question. This can be answered in the Ref. [16], where Udita N. Katugampola gave a general definition of the fractional derivative, which was used to define more generalized interesting fractional derivatives.

As physical applications of conformable fractional derivative [17–21], M.A. Hammad and R. Khalil [17] gave the solution for the conformable fractional heat equation and W.S. Chung [18] discussed fractional Newtonian mechanics; Mostafa Eslami [19] got exact traveling wave solutions to the fractional coupled nonlinear Schrödinger equations; A. Kurt, Y. Çenesiz,

☆ By the Natural Science Foundation of China (NSFC) under Grant (11371287 and 61663043).

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https://doi.org/10.1016/j.camwa.2018.01.025
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O. Tasbozan [20] studied the numerical solution of Burgers equation by using the homotopy analysis method (HAM); Olaniyi and Gbenga [21] obtained the exact solutions of Fornberg–Whitham equation with the new fractional derivative by applying q-homotopy analysis method (q-HAM). It is clearly seen that further studies and explanations can be made regarding the physical meaning and physical applications of this new subject area.

The aim of this paper is to find exact solutions of some time-fractional differential equations with conformable derivative, such as generalized Burgers equation, generalized KdV equation, generalized Sharma–Tasso–Olver (FSTO) equation, fifth-order KdV equation by using simplest equation method [22,23].

The rest of this paper is organized as follows. In Section 2, conformable fractional derivative and some properties are introduced briefly. Then the steps of simplest equation method are given in detail. In Section 3, we apply this method to generalized Burgers equation, generalized KdV equation, generalized Sharma–Tasso–Olver (FSTO) equation and fifth-order KdV equation, $(3+1)$-dimensional KdV–Zakharov-Kuznetsov (KdV–ZK) equation. Some conclusions are presented at the end of the paper.

2. Conformable fractional derivative and simplest equation method

2.1. Definition and properties of conformable fractional derivative

Now, we briefly introduce the definition and properties of conformable fractional derivative in Refs. [13–15,19].

Definition ([13–15,19]). Given a function $f : (0,\infty) \to \mathbb{R}$, then the conformable fractional derivative of f of order α is defined by

$$T_\alpha(f)(t) = \lim_{\varepsilon\to 0} \frac{f(t+\varepsilon t^{1-\alpha}) - f(t)}{\varepsilon}$$

for all $t > 0$, $\alpha \in (0, 1)$.

If the conformable fractional derivative of f of order α exists, then we simply say f is α-differentiable [13–15].

Properties ([13–15,19]). Let $\alpha \in (0, 1)$ and f, g be α-differentiable at a point $t > 0$, then some properties of the conformable fractional derivative are as follows:

(I) $T_\alpha(af + bg) = aT_\alpha(f) + bT_\alpha(g)$, $a, b \in \mathbb{R}$;
(II) $T_\alpha(t^\mu) = \mu t^{\mu-\alpha}$, $\mu \in \mathbb{R}$;
(III) $T_\alpha(\lambda) = 0$, for all constant function $f(t) = \lambda$;
(IV) $T_\alpha(fg) = fT_\alpha(g) + gT_\alpha(f)$;
(V) $T_\alpha(\frac{f}{g}) = \frac{gT_\alpha(f) - fT_\alpha(g)}{g^2}$;
(VI) If, in addition to f differentiable, then $T_\alpha(f)(t) = t^{1-\alpha}\frac{\mathrm{d}f}{\mathrm{d}t}$.

This definition also satisfies the chain rule as follows.

Theorem (*Chain Rule* [14,19]). *Assume functions f, $g : [0, \infty) \to \mathbb{R}$ be α-differentiable, where $(0 < \alpha \le 1)$. Then the following rule is obtained*

$$T_\alpha(f \circ g)(t) = t^{1-\alpha} g'(t) f'(g(t)).$$

2.2. Simplest equation method for time-fractional partial differential equations

We consider the following time-fractional partial differential equations

$$F(u, D_t^\alpha u, u_{x^i}, u_{x^i x^j}, u_{x^i x^j x^k}, \ldots) = 0, \tag{1}$$

where D_t^α is conformable fractional derivative and $\alpha \in (0, 1)$. Eq. (1) has $n+1$ independent variables $x = (x^1, x^2, \ldots, x^n)$ and one dependent variable $u = u(x, t)$, where

$$u_{x^i} = \frac{\partial u}{\partial x^i},\quad u_{x^i x^j} = \frac{\partial^2 u}{\partial x^i \partial x^j},\quad u_{x^i x^j x^k} = \frac{\partial^3 u}{\partial x^i \partial x^j \partial x^k}, \ldots.$$

We will apply simplest equation method [22,23] to find exact solutions of Eq. (1). Here we outline the main steps of simplest equation method.

Step 1 Applying the following traveling wave transformation

$$u(x,t) = U(\xi),\quad \xi = \sum_{i=1}^{n} l_i x_i + \frac{\omega}{\alpha} t^\alpha, \tag{2}$$