ARTICLE IN PRESS

Computers and Mathematics with Applications ▮ (▮▮▮▮) ▮▮▮–▮▮▮

Contents lists available at ScienceDirect

Computers and Mathematics with Applications

journal homepage: www.elsevier.com/locate/camwa

Solving complex PIDE systems for pricing American option under multi-state regime switching jump–diffusion model

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ARTICLE INFO

Article history:
Received 4 November 2017
Received in revised form 8 January 2018
Accepted 20 January 2018
Available online xxxx

Keywords:
Partial integral differential equations
L-stable methods
American options
Regime switching
Jump–diffusion

ABSTRACT

Based on exponential time differencing approach, an efficient second order method is developed for solving systems of partial integral differential equations. The method is implemented to solve American options under multi-state regime switching with jumps. The method is seen to be strongly stable (*L*-stable) and avoids any spurious oscillations caused by non-smooth initial data. The predictor–corrector nature of the method makes it highly efficient in solving nonlinear PIDEs in each regime with different volatilities and interest rates. Penalty method approach is applied to handle the free boundary constraint of American options. Numerical results are presented to illustrate the performance of the method for American options under Merton's jump–diffusion models. Padé approximation of matrix exponential functions and partial fraction splitting technique are applied to construct computationally efficient version of the method. Efficiency, accuracy and reliability of the method are compared with those of the existing methods available in the literature.

1. Introduction

The pricing option problems in regime-switching models have drawn considerable attention, see for example [1–3] and references therein. These models are capable of modeling non-constant and random market parameters, like, volatility and interest rate. The asset prices in these models are dictated by a number of stochastic differential equations which are coupled by a finite-state Markov chain representing randomly changing economical factors. Drift and volatility coefficients are assumed to depend on the Markov chain and are allowed to take different values in different regimes resulting in a situation where both continuous dynamics and discrete events are present.

American option under m_o regimes satisfies a system of m_o free boundary value problems. An (optimal) early exercise boundary is associated with each regime. The use of penalty approach results in a system of m_o coupled nonlinear partial differential equations in m_o states. Recently, Holmes et al. [4] developed a front-fixing finite element method for the valuation of American options with regime switching. However their approach is restricted to only two regimes. Khaliq et al. [5] generalized the idea of penalty term to regime-switching case by adding a penalty term to each of the m_o systems of PDEs which results in solving each system on a fixed rectangular domain.

Contrary to models with continuous paths, jump–diffusion models allow large sudden changes in the price of the underlying asset. The driving Brownian motion is a continuous process which makes it difficult to fit the market data with large fluctuations. Large market movements as well as a great amount of information arriving suddenly (i.e. a jump) lead to

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https://doi.org/10.1016/j.camwa.2018.01.026

the study of jump–diffusion models. In order to include the influence of macroeconomic factors on the behavior of individual asset prices, regime-switching models are considered in recent years, see [6]. In [7], Merton proposed to include jumps into the Black–Scholes model. The rational for including a jump component in a diffusion model is due to large market movements. A regime-switching jump–diffusion model proposed in [8] includes both jump and regime-switching in an appropriate way.

For pricing American options under regime-switching stochastic process, Huang et al. [9] analyze a number of techniques including both explicit and implicit discretizations. They compared a number of iterative procedures for solving the associated nonlinear algebraic equations. Their numerical experiments indicate that a fixed point policy iteration, coupled with a direct control formulation, is a reliable general purpose method.

Jump–diffusion model proposed by Merton [7] is considered in this paper. Unlike Black–Scholes model, Jump–diffusion models do not have closed form solution. Therefore, several numerical studies have been conducted for pricing options under jump–diffusion models. Alternating Directions Implicit (ADI) finite difference method combined with the discrete Fourier Transform DTF has been used by Andersen and Andreasen [10]. Whereas, Multinomial trees method was suggested by Amin [11] but this method is restricted by the number of time steps, and it is just of first order convergence. Almendral and Oosterlee in [12] proposed operator-splitting technique with iterative methods for European options. Implicit explicit (IMEX) finite difference method was proposed by Cont et al. [13] to avoid a full dense matrix inversion.

We present and analyze a strongly stable numerical method for solving the American option problems with multi-state regime switching jump–diffusion. It combines the penalty method [14] with an implicit predictor–corrector scheme and utilizes (0, 2)-Padé approximation to the matrix exponential functions which leads to a positivity preserving, strongly stable and reliable numerical method in each regime. These are essential tools to handle the problem due to nonsmooth payoff, see for example, Yousuf et al. [15]. Partial fraction splitting technique is applied to construct computationally efficient version of the method which can also be implemented in parallel.

We intend to numerically solve a partial integral differential equation (PIDE) arising in the jump–diffusion model. Numerical solutions of the coupled systems of nonlinear partial integral differential equations are obtained using the following steps. Free boundary value system is converted to a system over a fixed temporal domain using penalty method approach. Spatial discretization of the differential and integral operators converts the system of PIDEs to a system of ODEs. Exact solution of each ODE in the system is written using Duhamel's principle and L-stable predictor–corrector time stepping method is applied to approximate the solution. Although the method is applicable to many states regime switching problems, we implemented it to solve American put option and American butterfly option with four regimes under jump–diffusion model. We compare the results of the new method with some existing method in the literature. Numerical results are reported to illustrate the second order convergence in time.

This paper is organized as follows. In Section 2 we state the PIDE in regime-switching with jump–diffusion model for pricing American options. In Section 3 we describe the differential and integral operator discretization. Section 4 is devoted to the development of time stepping scheme, stability of the method and algorithm to implement the method. Numerical experiments are given in Section 5. American put option and American butterfly option problems are solved in this section. Convergence tables are given for the American put option at the strike price for each regime. Reliability of the method through out the time domain is shown by time evolution graphs. Efficiency of the method is also given in this section by comparing CPU time with an other method. Conclusion and future work direction is given in Section 6.

2. Regime-switching jump–diffusion model

We consider a continuous-time Markov chain α_t which takes values among m_o different states where each state represents a particular regime. The state space of α_t is given by $\mathcal{M} := \{1, \ldots, m_o\}$ and the matrix $Q = (q_{ij})_{m_o \times m_o}$ denotes the generator of α_t. It is assumed that Q is known and its entries q_{ij} satisfy the following:

(I) $q_{ij} \geq 0$ if $i \neq j$;

(II) $q_{ii} \leq 0$ and $q_{ii} = -\sum_{j \neq i} q_{ij}$ for each $i = 1, \ldots, m_o$.

See for example, Yin and Zhang [16].

Introducing a Markov chain α_t into the option pricing model will result in an incomplete market which implies that the risk-neutral measure is not unique. To determine a risk-neutral measure for option pricing, one can employ a regime-switching random Esscher transform [17]. Let the risk-neutral probability space $(\Omega, \mathcal{F}, \widetilde{\mathcal{P}})$ is given and let $\widetilde{B}_t$ be a standard Brownian motion defined on $(\Omega, \mathcal{F}, \widetilde{\mathcal{P}})$ and assume it is independent of the Markov chain α_t. We consider the following regime-switching geometric Brownian motion (GBM) for the risk-neutral process of the underlying asset price S:

$$\frac{dS}{S} = \mu_{\alpha_t} dt + \sigma_{\alpha_t} d\widetilde{B}_t + dJ_{\alpha_t}, \quad t \geq 0, \tag{1}$$

where σ_{α_t} is the volatility of the asset S and $\mu_{\alpha_t} = r_{\alpha_t} - D_\alpha - \lambda_{\alpha_t}\kappa$ is the drift rate for each regime with risk-free interest rate $r_{\alpha_t} \geq 0$, D_α is the continuous dividend yield. Since both σ_{α_t} and r_{α_t} are assumed to depend on the Markov chain α_t, they can