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An efficient split-step quasi-compact finite difference method for the nonlinear fractional Ginzburg–Landau equations ☆

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ABSTRACT

In this paper, we propose a split-step quasi-compact finite difference method to solve the nonlinear fractional Ginzburg–Landau equations both in one and two dimensions. The original equations are split into linear and nonlinear subproblems. The Riesz space fractional derivative is approximated by a fourth-order fractional quasi-compact method. Furthermore, an alternating direction implicit scheme is constructed for the two dimensional linear subproblem. The unconditional stability and convergence of the schemes are proved rigorously in the linear case. Numerical experiments are performed to confirm our theoretical findings and the efficiency of the proposed method.

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1. Introduction

The conventional complex Ginzburg–Landau equation (GLE) is studied widely in physics, chemistry and biology, which describes a lot of phenomena including nonlinear waves, second-order phase transitions, superfluidity and Bose–Einstein condensation [1,2]. The fractional Ginzburg–Landau equation (FGLE) as a generalization of the GLE is derived in [3–5] from the variational Euler–Lagrange equation for fractal media. Later, the FGLE has drawn attention of many researchers, such as a network of diffusively Hindmarsh–Rose neurons with long-range synaptic coupling in the infrared limit [6] and an application of the fractional derivative formalism to the thermodynamics of second-type phase transitions in the presence of a coexisting nonlocal ordering [7]. For theoretical studies, such as the well-posedness, long-time dynamics and asymptotic behaviors of FGLE, we refer to [8–10] and references therein.

In this paper, we consider the d-dimensional ($d = 1, 2$) FGLE with Riesz fractional derivative in space ($1 < \boldsymbol{\alpha_d} \le 2$)

$$u_t + (\upsilon + i\eta)L_{\mathrm{x}}^{\alpha_d} u + (\kappa + i\zeta)|u|^2 u - \gamma u = 0, \mathrm{x} \in R^d,\ 0 < t \le T, \tag{1.1}$$

$$u(\mathrm{x}, 0) = u_0(\mathrm{x}),\ \mathrm{x} \in R^d, \tag{1.2}$$

where $i^2 = -1$, $u(\mathrm{x}, t)$ is a complex-valued function, $\upsilon > 0, \kappa > 0$, η, ζ, γ are given real constants, u_0 is a given smooth function and $L_{\mathrm{x}}^{\alpha_d}$ denotes the Riesz fractional operator in space. In the one dimensional (1D) case, we denote

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$L_x^{\alpha_1}u=\frac{\partial^\alpha}{\partial|x|^\alpha}u(x,t)$, which is defined by

$$\frac{\partial^\alpha}{\partial|x|^\alpha}u(x,t)=-\frac{1}{2cos(\alpha\pi/2)\Gamma(2-\alpha)}\frac{d^2}{dx^2}\int_{-\infty}^{+\infty}(x-\xi)^{1-\alpha}u(\xi,t)d\xi,\ 1<\alpha\leq 2,$$

or, equivalently,

$$\frac{\partial^\alpha}{\partial|x|^\alpha}u(x,t)=-\frac{1}{2\cos\frac{\alpha\pi}{2}}[{}_{-\infty}D_x^\alpha u(x,t)+{}_xD_{+\infty}^\alpha u(x,t)], \tag{1.3}$$

where ${}_{-\infty}D_x^\alpha u(x,t)$ is the left Riemann–Liouville fractional derivative [11]

$$_{-\infty}D_x^\alpha u(x,t)=\frac{1}{\Gamma(2-\alpha)}\frac{d^2}{dx^2}\int_{-\infty}^{x}\frac{u(\xi,t)}{(x-\xi)^{\alpha-1}}d\xi, \tag{1.4}$$

and ${}_xD_{+\infty}^\alpha u(x,t)$ is the right Riemann–Liouville fractional derivative

$$_xD_{+\infty}^\alpha u(x,t)=\frac{1}{\Gamma(2-\alpha)}\frac{d^2}{dx^2}\int_{x}^{+\infty}\frac{u(\xi,t)}{(\xi-x)^{\alpha-1}}d\xi. \tag{1.5}$$

In the two dimensional (2D) case, we denote $L_x^{\alpha_2}u=\frac{\partial^\alpha}{\partial|x|^\alpha}u(x,y,t)+\frac{\partial^\beta}{\partial|y|^\beta}u(x,y,t)$, where $1<\alpha\leq 2$, $1<\beta\leq 2$, and $\frac{\partial^\alpha}{\partial|x|^\alpha}u(x,y,t)$ and $\frac{\partial^\beta}{\partial|y|^\beta}u(x,y,t)$ can be defined similarly. We mention that there exist several definitions for fractional derivatives in the literature, which may have different meanings (see [12–15] for more details). For example, the Riesz fractional derivative, which can be used to model anomalous diffusion, allows the modeling of flow regime impacts from either side of the domain [16]. The well-known fractional Laplacian is defined as a pseudo-differential operator with the symbols $-|\xi|^\alpha$:

$$-(-\Delta)^{\alpha/2}u(x)=-\mathscr{F}^{-1}(|\xi|^\alpha\mathscr{F}(u(\xi,t))), \tag{1.6}$$

where $\mathscr{F}$ denotes the Fourier transform. It is derived by the characteristic function of rotationally symmetric α-stable Lévy motion [13]. In the 1D case, the Riesz fractional operator (1.3) can be proved to be equivalent to the fractional Laplacian under a very mild condition (see [17]). In the 2D case, however, this equivalence is no longer true, except for some very special cases.

From the numerical point of view, there have been various numerical methods to solve the conventional GLEs, including spectral method [18,19], finite element method [20–22] and finite difference method [23–25] etc. Another related case is the fractional Schrödinger equation (FSE), which corresponds to Eq. (1.1) with $\upsilon=\kappa=\gamma=0$ and which has also received much attention [26–33]. Nevertheless, there is a little attention on the numerical solution of FGLE. Mvogo et al. [6] proposed a semi-implicit Riesz fractional finite difference scheme with accuracy of first-order in time and second-order in space. Wang and Huang [34] developed an implicit midpoint difference scheme and showed that the difference scheme has second-order accuracy both in time and space. Hao and Sun [35] derived a linearized high-order accurate difference scheme. The fractional centered difference scheme and compact technique were applied for the spatial approximation, and leap-frog scheme was used for the time approximation. The proposed scheme is of order $O(\tau^2+h^4)$ in maximum norm. Up to now, to the best of our knowledge, there is seldom numerical study for 2*D* FGLE.

To efficiently solve the nonlinearity problem, especially for high dimensional ones, it is necessary to construct efficient schemes. Many techniques have been used into classical GLE and FSE, such as linearized method [27,36] and split-step method [25,37]. In particular, split-step method has attracted more and more attentions. In the context of fractional version, Zhao et al. [36] proposed a compact alternating direction implicit (ADI) scheme for two dimensional FSE. Wang and Huang [37] used a split-step ADI method to solve nonlinear FSE in two dimension. Up to date, we have not found any published papers concerning split-step finite difference (SSFD) methods for the FGLE. In the current paper, we propose a split-step quasi-compact method for solving the space FGLE in 1D, which combine the SSFD for handling the nonlinearity with the fourth-order quasi-compact operator [38,39] to approximate the Riesz derivative. In order to reduce the computational cost, ADI technique is utilized to two dimension case. Therefore, we derive the split-step quasi-compact ADI (SSQCADI) scheme for the two dimensional problem.

The remainder of this paper is arranged as follows. In Section 2, we recall the quasi-compact approximation to the Riesz fractional derivative and some technical lemmas are presented. In Section 3, we construct the split-step method quasi-compact schemes for FGLE in one and two dimensions. Section 4 and Section 5 are devoted to rigorous theoretical analysis, including the boundedness and convergence of the proposed schemes for linear problem both in 1D and 2D. Numerical experiments are performed in Section 6 and some conclusions are drawn in Section 7.

2. Preliminaries

2.1. Quasi-compact WSGD operator for Riesz fractional derivatives

High-order approximations of the Riemann–Liouville fractional derivative have been considered by many authors and various numerical methods have been developed [39,40]. Here we adopt a fourth order quasi-compact weighed shifted

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