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Invariant analysis and conservation laws of (2 + 1) dimensional time-fractional ZK–BBM equation in gravity water waves

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ABSTRACT

This paper intends to make an in-depth study on the symmetry properties and conservation laws of the (2 + 1) dimensional time fractional Zakharov–Kuznetsov–Benjamin–Bona–Mahony (ZK–BBM) equation with Riemann–Liouville fractional derivative. Symmetry properties have been investigated here via Lie symmetry analysis method. In view of Erdélyi-Kober fractional differential operator, the reduction of (2 + 1) dimensional time fractional ZK–BBM equation has been done into fractional ordinary differential equation. To analyse the conservation laws, new theorem of conservation law has been proposed here for constructing the new conserved vectors for (2 + 1) dimensional time fractional ZK–BBM equation with the help of formal Lagrangian.

1. Introduction

We have considered here the (2 + 1) dimensional time-fractional ZK–BBM equation [1–3] in following form:

$$D_t^\alpha u + u_x + 2auu_x + b(u_{xt} + u_{yy})_x = 0, \tag{1.1}$$

where $0 < \alpha \leq 1$, a and b are constants.

The Zakharov–Kuznetsov–Benjamin–Bona–Mahony (ZK–BBM) equation is defined as the regularized long wave equation, which was initially derived under the assumption of small water wave amplitude $u(x, y, t)$ having large wavelength [4]. It also describes the wave water phenomena within the large wavelength limit and one special case of nonlinear reaction–diffusion equation known as the generalized Fisher equation. The ZK–BBM equation provides the description of gravity water waves, which is unidirectional propagation of long waves in certain nonlinear dispersive systems, propagates in the long-wave regime. Many observable physical phenomena like interaction of solitary waves and shock waves have been described by ZK–BBM equation [2,4].

Many researchers have tried in past to find the analytical solution of ZK–BBM equation. The methods like sine–cosine method [5], fractional sub-equation method [6,7], first integral method [8], Exp-function method [9], homotopy analysis method [10], modified simple equation method [11], exponential rational function method [12] have been used for finding the analytical solution of ZK–BBM equation.

The purpose of this paper is to use Lie symmetry analysis method [13–23] to obtain the similarity solutions and new conservation laws [24–26] of the (2 + 1) dimensional time fractional ZK–BBM equation. By using new theorem of

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conservation law, we have constructed the new conserved vectors for $(2+1)$ dimensional time fractional ZK–BBM equation with the help of formal Lagrangians.

The remainder of this paper is organized as follows: some theory of Riemann–Liouville derivative and the algorithm of the Lie symmetry method have been presented in Section 2 and Section 3 respectively. In Section 4, by using the proposed method, we discuss the symmetry analysis of Eq. (1.1). In Section 5, by using similarity variables, the reduction equations have been obtained and then solving some reduction equations, we obtain the similarity solutions of Eq. (1.1). Conservation laws of Eq. (1.1) have been discussed in Section 6. A brief conclusion is presented in Section 7.

2. Riemann–Liouville derivative

The fractional order Riemann–Liouville derivative of order $\alpha(>0)$ is defined as [27,28]

$$D_t^{\alpha} f(t) = \begin{cases} \dfrac{1}{\Gamma(m-\alpha)} \dfrac{d^m}{dt^m} \displaystyle\int_0^t (t-\tau)^{(m-\alpha-1)} f(\tau)\, d\tau & \text{if } m-1<\alpha<m, m \in \mathbb{N}, \\ \dfrac{d^m f(t)}{dt^m} & \text{if } \alpha = m,\ m \in \mathbb{N}, \end{cases} \tag{2.1}$$

and it has following property

$$D^{\alpha} t^{\gamma} = \frac{\Gamma(\gamma+1)\, t^{\gamma-\alpha}}{\Gamma(\gamma-\alpha+1)} \quad \gamma > \alpha - 1. \tag{2.2}$$

3. The general procedure of Lie symmetry analysis method for solving fractional partial differential equations

In this section, we carry out the brief detail of fractional Lie symmetry analysis for determining the symmetries for fractional nonlinear partial differential equations (FNPDEs).

Let us consider a FNPDE in the following form:

$$D_t^{\alpha} u = F(x, y, t, u, u_x, u_y, u_{xt}, u_{xy}, u_{yt}, \dots). \tag{3.1}$$

We now consider Eq. (3.1) is invariant under following one-parameter Lie group infinitesimal transformations acting on both the dependent and independent variables, given as

$$\begin{aligned} &\widetilde{x} \to x + \varepsilon\xi(x, y, t, u) + O(\varepsilon^2), \\ &\widetilde{y} \to y + \varepsilon\zeta(x, y, t, u) + O(\varepsilon^2), \\ &\widetilde{t} \to t + \varepsilon\tau(x, y, t, u) + O(\varepsilon^2), \\ &\widetilde{u} \to u + \varepsilon\eta(x, y, t, u) + O(\varepsilon^2), \\ &D_t^{\alpha}\widetilde{u} \to D_t^{\alpha} u + \varepsilon\eta_{\alpha}^{0}(x, y, t, u) + O(\varepsilon^2), \\ &\frac{\partial\widetilde{u}}{\partial\widetilde{x}} \to \frac{\partial u}{\partial x} + \varepsilon\eta^{x}(x, y, t, u) + O(\varepsilon^2), \\ &\frac{\partial^3\widetilde{u}}{\partial\widetilde{x}^2\partial\widetilde{t}} \to \frac{\partial^3 u}{\partial x^2\partial t} + \varepsilon\eta^{xxt}(x, y, t, u) + O(\varepsilon^2), \\ &\frac{\partial^3\widetilde{u}}{\partial\widetilde{x}\partial\widetilde{y}^2} \to \frac{\partial^3 u}{\partial x\partial y^2} + \varepsilon\eta^{xyy}(x, y, t, u) + O(\varepsilon^2) \\ &\dots \end{aligned} \tag{3.2}$$

where $\varepsilon \ll 1$ is a group parameter and ξ, ζ, τ, η are the infinitesimals of the transformations for the dependent and independent variables respectively. The explicit expressions of η^x, η^{xxt} and η^{xyy} are given by

$$\begin{aligned} &\eta^{x} = D_x(\eta) - u_x D_x(\xi) - u_y D_x(\zeta) - u_t D_x(\tau), \\ &\eta^{xx} = D_x(\eta^{x}) - u_{xx} D_x(\xi) - u_{xy} D_x(\zeta) - u_{xt} D_x(\tau), \\ &\eta^{y} = D_y(\eta) - u_x D_y(\xi) - u_y D_y(\zeta) - u_t D_y(\tau), \\ &\eta^{yy} = D_y(\eta^{y}) - u_{xy} D_y(\xi) - u_{yy} D_y(\zeta) - u_{yt} D_y(\tau), \\ &\eta^{xxt} = D_x(\eta^{xt}) - u_{xxx} D_x(\xi) - u_{xxy} D_x(\zeta) - u_{xxt} D_x(\tau), \\ &\eta^{xyy} = D_x(\eta^{yy}) - u_{xxy} D_x(\xi) - u_{xyy} D_x(\zeta) - u_{xyt} D_x(\tau), \\ &\dots \end{aligned} \tag{3.3}$$

where D_x, D_y and D_t are the total differentiation with respect to x, y and t respectively, which are defined for $x^1 = x$, $x^2 = y$ and $x^3 = t$ as

$$D_{x^i} = \frac{\partial}{\partial x^i} + u_i \frac{\partial}{\partial u} + u_{ij} \frac{\partial}{\partial u_j} + \cdots, \quad i, j = 1, 2, 3, \dots$$

and $u_i = \frac{\partial u}{\partial x^i}$, $u_{ij} = \frac{\partial^2 u}{\partial x^i \partial x^j}$ and so on.

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