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Gradient-based iterative algorithms for generalized coupled Sylvester-conjugate matrix equations☆

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ABSTRACT

By applying the hierarchical identification principle, the gradient-based iterative algorithm is suggested to solve a class of complex matrix equations. With the real representation of a complex matrix as a tool, the sufficient and necessary conditions for the convergence factor are determined to guarantee that the iterative solutions given by the proposed algorithm converge to the exact solution for any initial matrices. Also, we solve the problem which is proposed by Wu et al. (2010). Finally, some numerical examples are provided to illustrate the effectiveness of the proposed algorithms and testify the conclusions suggested in this paper.

1. Introduction

Lyapunov and Sylvester matrix equations play important roles in system theory, control theory and many applications of engineering problems [1–7]. For example, in stability analysis of linear jump systems with Markovian transitions, the following matrix equations are typical coupled Lyapunov matrix equations

$$A_i^T + P_iA_i + Q_i + \sum_{j=1}^{n} \pi_{ij}P_j = 0,\ i = 1, 2, \ldots, n, \tag{1.1}$$

where Q_i are positive definite matrices, π_{ij} are known transition probabilities and P_j are the unknown matrices [5,6]. In distributed parameter systems which characterize vibrating structures such as automobiles, bridges and buildings, we need to find the symmetric solutions of the Sylvester matrix equation [7]

$$AXB + CYD = E. \tag{1.2}$$

Although exact solutions, which can be computed by using the Kronecker product, are important, the computational efforts rapidly increase with the increasing of dimensions of the matrices to be solved. For some applications such as stability analysis, it is often not necessary to compute exact solutions, approximate solutions or bounds of solutions are sufficient. Moreover, if the parameters in system matrices are uncertain, it is not possible to obtain exact solutions for robust stability

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results [8–16]. Based on this fact, iterative algorithms have received much attention in the field of matrix algebra and system identification [17–19].

By extending the idea of CG method, Dehghan and Hajarian [20] considered the problem of finding a generalized centro-symmetric solution pair of the generalized coupled Sylvester matrix equations

$$\sum_{i=1}^{l} A_i X B_i + \sum_{i=1}^{l} C_i Y D_i = M \tag{1.3}$$

and

$$\sum_{i=1}^{l} E_i X F_i + \sum_{i=1}^{l} G_i Y H_i = N. \tag{1.4}$$

The generalized conjugate direction algorithm for solving the general coupled matrix equations over symmetric matrices was derived by Hajarian [21]. The matrix form of CGLS algorithm was given to solve linear matrix equations [22,23]. Finite iterative method is also an active method for solving the linear matrix equations [20–36]. For example, the least squares generalized Hamiltonian solution of the generalized coupled Sylvester-conjugate matrix equations was discussed in [24]. To simplify the proof process, several linear operators are used to prove the convergence of the conjugate gradient algorithm for solving a class of complex coupled matrix equations with conjugate transpose unknown matrices [26]. Applying finite iterative method, the solutions of generalized Sylvester-conjugate matrix equations were obtained [27–29]. The authors [25] derived the symmetric least squares solution of a class of Sylvester matrix equations via MINIRES algorithm. The finite iterative algorithm for solving the least-norm generalized (P, Q) reflexive solution of the matrix equations $A_i X B_i = C_i$ was studied [37]. Wang [38] established finite iterative algorithm for the generalized reflexive and anti-reflexive solutions of the linear matrix equation $AXB = C$.

The hierarchical identification principle plays an important role in system identification area [39–42]. Recently, this principle has been used to solve matrix equations. By applying the hierarchical identification principle, Ding and Chen presented the gradient-based and the least squares-based iterative algorithms for solving $Ax = b$, $AXB = F$ and other matrix equations [43–46]. For example, the gradient-based iterative algorithms were established for $AXB + CXD = F$, $AXB + CX^TD = F$ [47–50] and for the coupled matrix equations $A_1XB_1 = F_1$ and $A_2XB_2 = F_2$ [51,52]. Wu et al. applied gradient-based algorithms to solve the complex matrix equation $AXB + C\overline{X}D = F$ [53] and the extended coupled Sylvester-conjugate matrix equations [54] as well as a class of complex conjugate and transpose matrix equations [55]. Niu et al. [56] proposed the relaxed gradient-based iterative algorithm for solving Sylvester matrix equation. The optimal convergence factor of the gradient based iterative algorithm for linear matrix equations was discussed in [57]. Wang et al. [58] derived modified gradient based algorithm for solving Sylvester matrix equation.

Although some classic conclusions of the gradient-based and least squares-based iterative methods for solving linear matrix equations have been established systematically [57–63], there are still some problems need to be solved. For example, the gradient-based iterative algorithm for solving the following coupled Sylvester-conjugate matrix equations

$$\sum_{j=1}^{l} A_{ij} X_j B_{ij} + \sum_{j=1}^{l} C_{ij} \overline{X}_j D_{ij} = F_i,\ \ i = 1, 2, \ldots, s, \tag{1.5}$$

was established and the sufficient condition for the convergence of the algorithm was obtained [54]. Here, $A_{ij}, C_{ij} \in \mathbb{C}^{m_i \times r_j}$, $B_{ij}, D_{ij} \in \mathbb{C}^{t_j \times n_i}$ and $F_i \in \mathbb{C}^{m_i \times n_i}$, $i = 1, 2, \ldots, s, j = 1, 2, \ldots, l$, are given matrices and $X_j \in \mathbb{C}^{r_j \times t_j}$, $j = 1, 2, \ldots, l$, are unknown matrices to be determined. However, in Remark 2 of [54], Wu et al. pointed out that the range of the parameter μ given in Theorem 1 of [54] may be a bit conservative. In other words, they conjectured the gradient-based algorithm might be still convergent even if the parameter μ does not satisfy the condition of Theorem 1 in [54]. So far, this conjecture is still open. In this paper, by using the real representation of a complex matrix [53–55,64] and the vec-operator, we solve this problem and prove that the sufficient condition for the convergence of the gradient-based iterative algorithm (which has been established in Theorem 1 of [54]) for the system (1.5) is also the necessary condition. Also, by using the real representation of a complex matrix [65] and the vec operator, we derive another sufficient and necessary condition for the convergence of the gradient-based iterative algorithm for the system (1.5).

The rest of this paper is organized as follows. In Section 2, we give some preliminaries and related lemmas which will be used in this paper. Section 3 gives the new proof of the gradient-based iterative algorithm for solving the generalized coupled Sylvester-conjugate matrix equations (1.5). Section 4 offers the optimal convergence factor of the gradient-based iterative algorithm. In Section 5, the numerical experiments are given to show the efficiency of the proposed algorithms. Finally, we give our conclusions in Section 6.

2. Preliminaries

For convenience, we use the following notations throughout this paper. Let $\mathbb{R}^{m\times n}$ and $\mathbb{C}^{m\times n}$ be the sets of all real and complex $m \times n$ matrices, respectively. For $A \in \mathbb{C}^{n\times n}$, we write $\overline{A}$, A^T, A^H, A^{-1}, $\|A\|$ and $\|A\|_2$, $\sigma_{\max}(A)$, $\sigma_{\min}(A)$, $\rho(A)$, $\lambda(A)$, $\mathrm{rank}(A)$ to denote conjugation, transpose, the conjugate transpose, the inverse, Frobenius norm, the spectral norm, the maximal