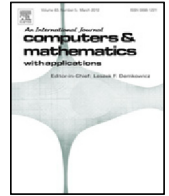




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Inhomogeneous Dirichlet boundary condition in the *a posteriori* error control of the obstacle problem

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ABSTRACT

We study *a posteriori* error control of finite element approximation of the elliptic obstacle problem with nonhomogeneous Dirichlet boundary condition. The results in the article are two fold. Firstly, we address the influence of the inhomogeneous Dirichlet boundary condition in residual based *a posteriori* error control of the elliptic obstacle problem. Secondly by rewriting the obstacle problem in an equivalent form, we derive *a posteriori* error bounds which are in simpler form and efficient. To accomplish this, we construct and use a post-processed solution \tilde{u}_h of the discrete solution u_h which satisfies the exact boundary conditions sharply although the discrete solution u_h may not satisfy. We propose two post processing methods and analyze them, namely the harmonic extension and a linear extension. The theoretical results are illustrated by the numerical results.

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1. Introduction

The elliptic obstacle problem is a popular prototype model for the study of elliptic variational inequalities. The applications of variational inequalities are enormous in the modern scientific computing world, e.g. in contact mechanics, option pricing and fluid flow problems. The numerical analysis of these classes of problems is an interesting subject as they offer challenges both in theory and computation. We refer to the books [1–4] for the theory of variational inequalities and their corresponding numerical analysis. Apart from these, we refer to the articles [5,6] and the recent articles [7–11] for the convergence analysis of finite element methods for the obstacle problem. One of the interesting properties that the obstacle problem exhibits is the free boundary along which the regularity of the solution is affected. It is worth remarking here that the location of free boundary is not known *a priori*. Adaptive finite element methods based on reliable and efficient *a posteriori* error estimates are of particular interest in this contest as they can capture the free boundaries by local mesh refinement around them. In designing any adaptive scheme, the first step is to derive some computable error estimators which are both reliable and efficient, see [12] for the analysis of *a posteriori* error control. There are many works in deriving residual based *a posteriori* error estimates for the obstacle problem, see [13–24] and see [25–28]. In recent years, much of research is focused on proving the convergence of adaptive methods based on *a posteriori* error estimates. In this direction, we refer to [29–34] for the work related to the obstacle problem. Further, we refer [35–38] and [39–42] for the work related to the numerical approximation of the Signorini contact problem.

In many occasions, it is assumed for the convenience in *a posteriori* error analysis of obstacle problems that the Dirichlet data is either zero or trace of a finite element function. However it is not clear if the error estimator with homogeneous Dirichlet boundary condition is reliable and efficient in the energy norm up to some Dirichlet data oscillations. The answer

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