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Drone like dynamics of dromion pairs in the (2+1) AKNS equation

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ABSTRACT

We employ Truncated Painlevé Approach (TPA) to the (2+1) dimensional AKNS equation and construct the solutions in closed form in terms of lower dimensional arbitrary functions of space and time. The highlight of our investigation is that we are able to generate dromions undergoing inelastic and elastic collisions. We observe that the conventional dromions undergo inelastic collision not only exchanging their energy, but also their phase while the dromion pair undergoes elastic collision. In particular, we observe that, we are able to turn ON or OFF the dynamic property of dromion pair by selectively choosing the lower dimensional arbitrary functions with a suitable initial condition. Similar to "drones", Unmanned Aerial Vehicles (UAVs), dromion pairs can be driven anywhere in the two dimensional plane by selectively giving the initial conditions. In addition to dromions, we have also generated a wide class of localized solutions such as rogue waves and lumps. We observe that while the rogue waves are found to be unstable and stationary, lumps do not interact with other, when they travel in the two dimensional plane.

1. Introduction

Rogue waves, also known as Freak waves occur in deep ocean [1–4]. The important feature of rogue waves is that they "come from nowhere and disappear with no trace" i.e. they appear for a short duration of time, like a shock wave. Recently, Rogue waves have received much attention in many fields such as hydrodynamics [5], Nonlinear optics [6–8], Bose Einstein Condensates [9,10], Plasma Physics [11], etc. In contrast to the Rogue waves, there is another interesting class of solutions which occur in (2+1) dimensional integrable systems, known as dromions [12–18]. They originate at the cross point of the intersection of two nonparallel ghost solitons, decay exponentially in all directions and are driven by lower dimensional boundaries or velocity potentials. There exists another class of localized solutions called lumps [19,20] which decay algebraically and do not interact with each other. The quest towards unearthing localized solutions in (2+1) dimensional nonlinear partial differential equations (pdes) with exotic behaviour continues even today.

In this paper, we consider the (2+1) dimensional AKNS equation [21] and employ Truncated Painlevé Approach (TPA) [22–26] to construct its solutions in closed form in terms of lower dimensional arbitrary functions of space and time. For a similar (2+1) dimensional Nonlinear equation [27], Lax pair is obtained using Singular Manifold method and Solitonic solutions such as Line solitons and dromions are constructed using Darboux Transformation. Even though the (2+1)

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dimensional AKNS equation has been investigated earlier and localized excitations like dromion lattice, solitoff and ring solitons have been constructed, their interaction (or collisional) dynamics has not yet been understood. The closed form of the solutions generated in this paper not only enables us to generate localized solutions like dromions, dromion pair, rogue waves and lumps, but also study their interactions. The highlight of our investigation is that the dromions are shown to exhibit both inelastic and elastic collisions. Dromions undergoing inelastic collision exchange not only their energy, but also their phase. On the other hand, in the elastic collision exhibited by dromion pair, we are able to turn ON or OFF the dynamical behaviour by selectively choosing the lower dimensional arbitrary functions of space and time combined with desirable initial conditions. This paper is organized as follows: In Section 2, we show, how the (2 + 1) dimensional AKNS equation can be solved using the TPA and the solution obtained in terms of the lower dimensional arbitrary functions of space and time. Section 3 is devoted to the construction of localized excitations, namely, dromions in particular and their collisional dynamics by suitably harnessing the arbitrary functions. We have also dwelt on the construction of rogue waves and lumps. Finally in Section 4, we summarize our results.

2. Truncated Painlevé approach

We now consider the (2+1) dimensional AKNS equation of the following form,

$$iu_t + u_{xx} + uL_x = 0, \tag{1}$$

$$L_y + |u|^2 = 0. \tag{2}$$

By taking $u = p$ and $u^* = q$, Eqs. (1) and (2) can be rewritten as follows,

$$ip_t + p_{xx} + pL_x = 0, \tag{3}$$

$$-iq_t + q_{xx} + qL_x = 0, \tag{4}$$

$$L_y + pq = 0. \tag{5}$$

By truncating the Laurent series of the solutions of Eqs. (3)–(5) at the constant level term, one obtains the following Bäcklund transformation,

$$p = \frac{p_0}{\phi} + p_1, q = \frac{q_0}{\phi} + q_1, L = \frac{L_0}{\phi} + L_1. \tag{6}$$

Assuming the following seed solution:

$$p_1 = q_1 = 0; L_1 = L_1(x, t), \tag{7}$$

we now substitute Eq. (6) with the above seed solution given by Eq. (7) into Eqs. (3)–(5) and collect the coefficients of $(\phi^{-3}, \phi^{-3}, \phi^{-2})$, to obtain,

$$-L_0p_0 + 2p_0\phi_x^2 = 0, \tag{8}$$

$$-L_0q_0 + 2q_0\phi_x^2 = 0. \tag{9}$$

$$p_0q_0 - L_0\phi_y = 0. \tag{10}$$

By solving Eqs. (8)–(10), we get,

$$L_0 = 2\phi_x, \tag{11}$$

$$p_0q_0 = 2\phi_x\phi_y. \tag{12}$$

Collecting the coefficients of $(\phi^{-2}, \phi^{-2}, \phi^{-1})$, we get

$$-ip_0\phi_t + p_0L_{0x} - 2p_{0x}\phi_x - p_0\phi_{xx} = 0, \tag{13}$$

$$iq_0\phi_t + q_0L_{0x} - 2q_{0x}\phi_x - q_0\phi_{xx} = 0, \tag{14}$$

$$L_{0y} = 0. \tag{15}$$

Using Eqs. (11), (12) in Eqs. (13) and (14), the variables p_0 and q_0 can be solved as

$$p_0 = F(y, t)\exp\left[\frac{1}{2}\int\frac{-i\phi_t + \phi_{xx}}{\phi_x}dx\right], \tag{16}$$

$$q_0 = F(y, t)\exp\left[\frac{1}{2}\int\frac{i\phi_t + \phi_{xx}}{\phi_x}dx\right], \tag{17}$$