Computers and Mathematics with Applications ▮ (▮▮▮▮) ▮▮▮–▮▮▮

Contents lists available at ScienceDirect

Computers and Mathematics with Applications

journal homepage: www.elsevier.com/locate/camwa

An efficient two-step iterative method for solving a class of complex symmetric linear systems☆

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ARTICLE INFO

Article history:
Received 24 October 2017
Received in revised form 7 December 2017
Accepted 22 December 2017
Available online xxxx

MSC:
65F10
65F50

Keywords:
Complex symmetric linear systems
Two-step parameterized iteration method
Convergence properties
Inexact implementation
Preconditioning

ABSTRACT

In this paper, a new two-step iterative method called the two-step parameterized (TSP) iteration method for a class of complex symmetric linear systems is developed. We investigate its convergence conditions and derive the quasi-optimal parameters which minimize the upper bound of the spectral radius of the iteration matrix of the TSP iteration method. Meanwhile, some more practical ways to choose iteration parameters for the TSP iteration method are proposed. Furthermore, comparisons of the TSP iteration method with some existing ones are given, which show that the upper bound of the spectral radius of the TSP iteration method is smaller than those of the modified Hermitian and skew-Hermitian splitting (MHSS), the preconditioned MHSS (PMHSS), the combination method of real part and imaginary part (CRI) and the parameterized variant of the fixed-point iteration adding the asymmetric error (PFPAE) iteration methods proposed recently. Inexact version of the TSP iteration (ITSP) method and its convergence properties are also presented. Numerical experiments demonstrate that both TSP and ITSP are effective and robust when they are used either as linear solvers or as matrix splitting preconditioners for the Krylov subspace iteration methods and they have comparable advantages over some known ones for the complex symmetric linear systems.

1. Introduction

We consider the system of linear equations of the form:

$$Ax \equiv (W + iT)x = b, \tag{1}$$

where $W, T \in \mathbb{R}^{n\times n}$ are symmetric positive semi-definite matrices with at least one of them, for example, W, being positive definite, $b \in \mathbb{R}^n$ and $i = \sqrt{-1}$. Such systems widely arise from many applications in scientific and engineering computing. For background information on these kinds of the problems in scientific and engineering applications, see, e.g., [1–6].

A large number of iteration methods have been proposed for solving the linear system (1) in the recent literatures. Based on the Hermitian and skew-Hermitian splitting (HSS) of the coefficient matrix of (1): $A = H + S$, where $H = \frac{1}{2}(A + A^*) = W$ and $S = \frac{1}{2}(A - A^*) = iT$, Bai et al. [7] initially established the HSS iteration method for solving the non-Hermitian positive definite linear systems. Here, A^* denotes the conjugate transpose of the matrix A. However, at each iteration step of the HSS

☆ This research was supported by the National Natural Science Foundation of China (No. 11171273) and Innovation Foundation for Doctor Dissertation of Northwestern Polytechnical University (No. CX201628).

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https://doi.org/10.1016/j.camwa.2017.12.026
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iteration method, we need to solve a shift skew-Hermitian linear system. To overcome this difficult, Bai et al. [1] skillfully designed a modified HSS (MHSS) method and its preconditioned version which is called the PMHSS iteration method [2].

The PMHSS iteration method: Let $\alpha > 0$ be a positive constant and $V \in \mathbb{R}^{n\times n}$ be a symmetric positive definite matrix. Given an initial guess $x^{(0)}$. For $k = 0, 1, 2, \ldots,$ until $x^{(k)}$ converges, compute

$$\begin{cases} (\alpha V + W)x^{(k+\frac{1}{2})} = (\alpha V - iT)x^{(k)} + b, \\ (\alpha V + T)x^{(k+1)} = (\alpha V - iW)x^{(k+\frac{1}{2})} - ib. \end{cases} \quad (2)$$

In order to improve the convergence rate of the PMHSS iteration method, many researchers have developed some efficient iteration methods recently. In [8], Li et al. proposed the lopsided PMHSS (LPMHSS) iteration method and showed that the lPMHSS iteration method performs better than the PMHSS one when the real part of A is dominant. In the sequal, Zeng and Ma [9] derived a one-step iteration method for solving the system (1) which is named as the parameterized single-step HSS (PSHSS) iteration method and based on the SHSS one put forward by Li and Wu [10]. Then the PSHSS iteration method was generalized to the parameterized single-step preconditioned variant of HSS (PSPHSS) iteration method by Xiao et al. [11] which contains the PSHSS one as a special case.

Note that the HSS-like iteration methods for complex linear systems have been extended in many literatures, see [12,13], and some of them have been used for solving other systems of equations, like Sylvester equations, refer to [14–19] for more details.

Recently, Hezari et al. [4] designed a scale-splitting (SCSP) iteration method by multiplying a complex number $(\alpha - i)$ through both sides of the complex system (1) and proved that it is convergent to the unique solution of the linear system (1) for a loose restriction on the iteration parameter α. Moreover, Zheng et al. [20] combined the idea of symmetry of the PMHSS method and the technique of scaling to reconstruct complex linear system (1) to propose a double-step scale splitting (DSS) iteration method, which is proved to be unconditionally convergent, and converges faster than the PMHSS iteration method. Subsequently, by combining real and imaginary parts of A in (1), Wang et al. [21] derived the combination method of real part and imaginary part which is simply called the CRI iteration method as follows:

The CRI method: Let $\alpha > 0$ be a positive constant. Given an initial guess $x^{(0)}$. For $k = 0, 1, 2, \ldots,$ until $x^{(k)}$ converges, compute

$$\begin{cases} (\alpha T + W)x^{(k+\frac{1}{2})} = (\alpha - i)Tx^{(k)} + b, \\ (\alpha W + T)x^{(k+1)} = (\alpha + i)Wx^{(k+\frac{1}{2})} - ib. \end{cases} \quad (3)$$

They proved that the upper bound of the spectral radius of the CRI iteration method is smaller than that of the PMHSS one. Very recently, Xiao and Wang [22] developed a new single-step iteration method called the parameterized variant of the fixed-point iteration adding the asymmetric error (PFPAE) iteration method for solving (1). They analyzed the convergence properties of the PFPAE iteration method and derived its quasi-optimal parameters. Numerical results showed that this new method outperforms than some existing ones.

The PFPAE iteration method: Let $\alpha > 0$ and $\omega > 0$ be two positive constants. Given an initial guess $x^{(0)}$. For $k = 0, 1, 2, \ldots,$ until $x^{(k)}$ converges, compute

$$(\omega W + T)x^{(k+\frac{1}{2})} = [(1 - \alpha)(\omega W + T) - i\alpha(\omega T - W)]x^{(k)} + \alpha(\omega - i)b. \quad (4)$$

On the other hand, to avoid complex arithmetic, let $x = u + iv$ and $b = p + iq$ with $u, v, p, q \in \mathbb{R}^n$, then system (1) is equivalent to a 2-by-2 block real linear system

$$\begin{pmatrix} W & -T \\ T & W \end{pmatrix} \begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} p \\ q \end{pmatrix}, \quad (5)$$

with u and v being unknown vectors, which can be formally regarded as a special case of the generalized saddle point problem and was introduced in [23] more recently. For the classical saddle point problems, Bai et al. [24] established the generalized successive overrelaxation (GSOR) iteration method, and proved its convergence under suitable restrictions on the iteration parameters and determined its optimal iteration parameters. After that, Bai and Wang [25] proposed the inexact variants of the GSOR iteration method, which contain the parameterized inexact Uzawa (PIU) iteration method, and they studied the convergence properties of the PIU method. In particular, they discussed its quasi-optimal iteration parameters and the corresponding quasi-optimal convergence factor for the saddle point problems. Recently, Salkuyeh et al. [26] applied the GSOR iteration method to the system (5), and derived its convergence condition and optimal parameter. In order to further improve the efficiency of the GSOR iteration method, Hezari et al. [5] designed the preconditioned GSOR (PGSOR) iteration method for (5), they proved that the minimum value of the spectral radius of the PGSOR iteration method is smaller than that of the GSOR one. After that, Liang and Zhang [27] developed the symmetric SOR (SSOR) iteration method and its accelerated variant for (5), and their optimal iteration parameters were also obtained. For more iteration methods for linear systems (1) or (5), we can refer to References [3,23,28–35].

To further accelerate the convergence rates of the PFPAE and the DSS iteration methods for solving the system (1), we use the idea of PFPAE method and the technique of scaling to reconstruct complex linear system (1), but twice, to design