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An improved immersed finite element particle-in-cell method for plasma simulation☆

Jinwei Bai [a], Yong Cao [a], Yuchuan Chu [a,b,*], Xu Zhang [c]

[a] Department of Mechanical Engineering & Automation, Harbin Institute of Technology, Shenzhen Graduate School, Shenzhen, Guangdong 518055, PR China

[b] Department of Mathematics & Statistics, Missouri University of Science & Technology, Rolla, MO 65401, USA

[c] Department of Mathematics & Statistics, Mississippi State University, Mississippi State, MS 39762, USA

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ABSTRACT

The particle-in-cell (PIC) method has been widely used for plasma simulation, because of its noise-reduction capability and moderate computational cost. The immersed finite element (IFE) method is efficient for solving interface problems on Cartesian meshes, which is desirable for the PIC method. The combination of these two methods provides an effective tool for plasma simulation with complex interface/boundary. This paper introduces an improved IFE–PIC method that enhances the performance in both IFE and PIC aspects. For the electric field solver, we adopt the newly developed partially penalized IFE method with enhanced accuracy. For PIC implementation, we introduce a new interpolation technique to ensure the conservation of the charge. Numerical examples are provided to demonstrate the features of the improved IFE–PIC method.

1. Introduction

There are two classes of methods for plasma simulation. The first one is the traditional dynamic simulation [1], which is mainly used to obtain the distribution functions of particles by solving the Vlasov equation. The second one is the particle simulation method [2–4], which is used to track a large number of individual particles and to obtain the trajectory parameters and characteristics of plasma by statistical methods. Due to the enormous number of particles to be tracked and the limited computational resources, the development of the particle simulation method was quite slow. The particle simulation method entered a rapid developing period [5–7] since Birdsall and Langdon [8] introduced the particle-in-cell (PIC) method which utilizes the finite-sized particle (or cloud) instead of a huge number of real particles.

Immersed finite element (IFE) method is a finite element method for solving interface problems on uniform Cartesian meshes, which was first proposed by Li et al. [9]. Different from classical finite element methods using body-fitted meshes, the mesh of the IFE method is independent of the interface. However, the IFE basis functions around interfaces are modified to accommodate the interface jump conditions. The advantage of the IFE method is that structured Cartesian meshes can be used to solve interface problems with arbitrary interface geometry. For problems with a moving interface, IFE methods are especially advantageous since there is no need to regenerate the solution meshes repeatedly [10–12]. The IFE methods

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* Corresponding author at: Department of Mechanical Engineering & Automation, Harbin Institute of Technology, Shenzhen Graduate School, Shenzhen, Guangdong 518055, PR China.

E-mail address: ychuan.chu@hit.edu.cn (Y. Chu).

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have been developed for solving the second-order elliptic equations [13–17], elasticity equations [18–20], and Stokes equations [21], to name only a few.

In the past decade, the IFE method has been used together with the PIC method for plasma simulations [22–25]. The IFE method used as an electric field solver is performed on well-structured Cartesian meshes. This is particularly desirable for the PIC method because tracking a large number of plasma particles can be efficiently done in a uniform structured mesh. We refer to a few recent applications of the IFE–PIC method for different types of particle simulations, such as ion thruster [26,27], hall thruster [28], and lunar surface environment [29,30]. Also, the IFE–PIC method has been extended to handle unbounded interface problems with asymptotic boundary condition [31] and periodic boundary condition [32].

For the current IFE–PIC method, we noticed that there are two issues. First, the classical Galerkin IFE method is used as the field solver. As shown in [33], the classical IFE method is not accurate around the interface, because the IFE basis functions are discontinuous across the element boundaries, and the classical Galerkin formulation cannot control such discontinuity. Second, the particle interpolation method of the PIC algorithm is imperfect. The conventional interpolation approach applied on the interface element often leads to the non conservation of charge, because it neglects the fact that some nodes of the interface cells are inside the conductors. Similar problems occur in applying the electric field force to the particles on the interface elements.

In this paper, we introduce an improved IFE–PIC method that focuses on overcoming the problems mentioned above. As a remedy of discontinuity of IFE field solver, we adopt the newly developed partially penalized immersed finite element (PPIFE) method [33] to improve the accuracy of IFE methods near interfaces. For PIC interpolation, we introduce a two-step approach for particle interpolation that preserves the charge conservation. Comparing with conventional charge distribution in PIC, we add a correction step that redistributes the quantity distributed to the nodes inside the conductor to the remaining nodes in order to maintain charge conservation. In addition, we use IFE basis functions to calculate the electric field and force on the interface elements. The new approach can calculate the motion of particles more accurately.

The rest of the article is organized as follows. In Section 2, we recall the classical IFE method and the PIC interpolation. In Section 3, we present our improved IFE–PIC method. The improvement in the IFE solver part is the PPIFE method with additional penalty terms. The improvement in the PIC part includes the new particle interpolation scheme and the new method for the force deposit. In Section 4, we present some numerical experiments to compare the performance of traditional IFE–PIC method and improved IFE–PIC method. Brief conclusions will be given in Section 5.

2. Review of IFE–PIC method

In this section, we first recall the main steps in a typical IFE–PIC computational cycle. Then we will recall the classical IFE method and PIC interpolation that are widely used in the literature.

2.1. Main steps of IFE–PIC method

Real plasma particles are modeled as many macro-particles in the PIC method, and they follow the evolution of the orbits of individual particles in the self-consistent electromagnetic field. The field is then updated by solving the governing elliptic equation with discontinuous dielectric coefficients. The IFE–PIC method is an iteration of solving for the electromagnetic field and particle motion until the steady state is achieved.

In general, an IFE–PIC computational cycle consists of the following five steps:

Step 1. Initialization A series of initial settings of the simulation including domain, mesh, boundary condition, and initial position and velocity of particles are set up.

Step 2. Particle push The motion of the particle is induced by the particles themselves and the external fields $\boldsymbol{E}$. The trajectory of an individual charged particle is obtained by integrating the Newton–Lorentz equation

$$m\frac{d\mathbf{v}}{dt} = q(\boldsymbol{E} + \mathbf{v} \times \boldsymbol{B}), \tag{1}$$

where m, q and $\mathbf{v}$ are the mass, the charge, and the velocity of the particle, respectively. $\boldsymbol{B}$ denotes the static magnetic field.

Step 3. Charge deposit The change of the positions of particles leads to the change of the charge density ρ on each node. Thus, we need to calculate the charge density at each node according to the new positions of particles. The process of interpolating the particle charges on the discrete mesh points is called weighting. In traditional PIC method, we use the ratio of the area of the rectangle formed by the opposite cell vertex and the particle to the area of element as the weighting.

Step 4. Solving for potential After obtaining the charge density ρ on each node, the electric field should also be updated. The potential function $\Phi(\mathbf{x})$ can be described by the second-order Poisson's equation with discontinuous dielectric coefficient $\beta(\mathbf{x})$, which represents different types of material:

$$-\nabla \cdot (\beta \nabla \Phi) = \rho(\mathbf{x}). \tag{2}$$

To solve this equation, we use the IFE method as a field solver, because of its applicability of Cartesian mesh, which is desirable in the PIC simulation for fast tracking of particles' locations.

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