Computers and Mathematics with Applications ▮ (▮▮▮▮) ▮▮▮–▮▮▮

Contents lists available at ScienceDirect

Computers and Mathematics with Applications

journal homepage: www.elsevier.com/locate/camwa

Non-overlapping domain decomposition algorithm based on modified transmission conditions for the Helmholtz equation

Yassine Boubendir *, Dawid Midura

New Jersey Institute of Technology, Department of Mathematical Sciences, University Heights, Newark NJ 07102, USA

ARTICLE INFO

Article history:
Available online xxxx

Keywords:
Domain decomposition methods
Helmholtz equation
Transmission conditions

ABSTRACT

A square-root based transmission conditions domain decomposition method was recently introduced for the Helmholtz equation. It produces an effective algorithm where the convergence is independent of the wavenumber and the mesh discretization. We modify here these conditions in order to guarantee well-posedness of local problems and further improve the efficiency of the whole method. Numerical results, in particular in the three dimensional case, show significant reduction of the computational time needed in the iterative procedure while preserving the iteration number when compared with the original algorithm.

1. Introduction

In this paper, we are interested in non-overlapping Domain Decomposition Methods (DDMs) for the Helmholtz equation. Specifically, our focus resides on these approaches based on Robin boundary conditions [1–4]. In this framework, several efficient methods were developed for improving the convergence of the iterative procedure by modifying the original transmission conditions [5–17].

In [17], the authors introduced a new square-root based transmission conditions combined with a Padé approximants localization procedure that accurately approximates the Dirichlet-to-Neumann (DtN) operator. This strategy allows the design of algorithms with quasi-optimal convergence properties. Indeed, it is shown that the rate of convergence is optimal on the evanescent modes and is significantly improved for the remaining ones. In the effective convergence, this results in a DDM independent of the wavenumber as well as of the mesh discretization. Moreover, these techniques are easy to implement in a basic finite element solver. However, to our best knowledge, there is no proof of the well-posedness of the local problems. In addition, the utilization of the Padé approximants requires the resolution of a series of partial differential equations, on each artificial interface, coupled to each local problem. This increases the size of the related local linear systems, to be solved at each iteration, since it grows with the order of the Padé series. To overcome these difficulties, we introduce a new family of transmission conditions that we adapt to the case of Padé approximants approach. The main idea consists of modifying the boundary conditions so that the computation of the partial differential equations mentioned above is only done at the level of the iterates, located on the right hand side of the local problems and corresponding to the iterative procedure, which results in keeping the size of the local linear systems independent of the Padé number. The numerical analysis of the new algorithm shows that the method achieves the same accuracy with a significant reduction of the computational time when compared to the quasi-optimal algorithm [17], in particular in the three dimensional case, while preserving comparable number of iterations. Furthermore, these new transmission conditions can be generalized to other kind of operators.

* Corresponding author.
E-mail addresses: boubendi@njit.edu (Y. Boubendir), midurad@gmail.com (D. Midura).

http://dx.doi.org/10.1016/j.camwa.2017.07.027

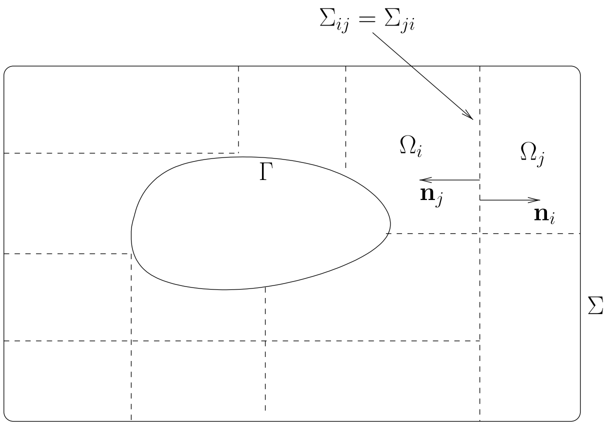

Fig. 1. Example of 2D non-overlapping domain decomposition method.

The paper is organized as follows. In Section 2, we introduce the scattering problem and the non-overlapping domain decomposition method. We present in Section 3 a non-local square-root operator which approximates the exact DtN transmission operator as well as its localization using Padé approximants technique. Section 4 develops the new modified transmission conditions and describes how to adapt the Padé approximants approach. Section 5 presents the finite element implementation of the resulting DDM. Section 6 is reserved for both two- and three-dimensional numerical results.

2. Scattering problem and non-overlapping domain decomposition method

Let us consider the three-dimensional time-harmonic scattering problem of an incident acoustic wave by an obstacle K. We want to compute the scattered field u solution to the exterior Helmholtz equation with a Dirichlet boundary condition[1]:

$$\begin{cases} \Delta u + k^2 u = 0 \quad \text{in } \mathbb{R}^3 \setminus K, \\ u = f \quad \text{on } \Gamma = \partial K, \\ \lim\limits_{|\mathbf{x}|\to\infty} |\mathbf{x}|(\partial_{|\mathbf{x}|} u - \imath k u) = 0, \end{cases} \tag{1}$$

where the data f is given in function of a plane wave: $f(\mathbf{x}) = -e^{\imath k \boldsymbol{\alpha}\cdot\mathbf{x}}$, with $\mathbf{x} = (x_1, x_2, x_3) \in \mathbb{R}^3$ and $\imath = \sqrt{-1}$. The incidence angle $\boldsymbol{\alpha}$ is normalized on the unit sphere ($|\boldsymbol{\alpha}| = 1$) and k denotes the wavenumber related to the wavelength λ of the incident wave through $k = 2\pi/\lambda$. The last equation of system (1) is the Sommerfeld radiation condition at infinity, which imposes that the scattered wave is outgoing. To solve (1), we combine Absorbing Boundary Conditions (ABCs) [18,19] with Lions–Després non-overlapping domain decomposition method [1–4]. The ABC method consists in truncating the infinite domain by introducing a fictitious boundary Σ to get a bounded computational region. Then, problem (1) is approximated by

$$\begin{cases} \Delta u + k^2 u = 0 \quad \text{in } \Omega, \\ u = f \quad \text{on } \Gamma, \\ \partial_{\mathbf{n}} u + \mathcal{T} u = 0 \quad \text{on } \Sigma, \end{cases} \tag{2}$$

where Ω is the bounded domain enclosed by the fictitious boundary Σ and Γ (see Fig. 1) and where the operator $\mathcal{T}$ represents an approximation of the DtN operator (for example $\mathcal{T} = -\imath k$) on Σ. The vector $\mathbf{n}$ is the outwardly directed unit normal to Σ. Let us remark that all what follows can be adapted to the case of other truncation techniques, like e.g., perfectly matched layers (PMLs) [20–23].

We are interested in the Lions–Després type domain decomposition method [1,4] that consists in splitting Ω into several subdomains Ω_i, $i = 1, \ldots, N_{\text{dom}}$, such that (see Fig. 1):

- $\overline{\Omega} = \bigcup_{i=1}^{N_{\text{dom}}} \overline{\Omega}_i$ $(i = 1, \ldots, N_{\text{dom}})$,
- $\Omega_i \cap \Omega_j = \emptyset$, if $i \neq j$, $(i, j = 1, \ldots, N_{\text{dom}})$,
- $\partial\Omega_i \cap \partial\Omega_j = \overline{\Sigma}_{ij} = \overline{\Sigma}_{ji}$ $(i, j = 1, \ldots, N_{\text{dom}})$ is the artificial interface separating Ω_i and Ω_j as long as its interior Σ_{ij} is not empty.

[1] The Dirichlet boundary condition models a sound-soft obstacle; Neumann (sound-hard) or Fourier–Robin (impedance) boundary conditions may also be set, which would not fundamentally change the rest of the paper.

Please cite this article in press as: Y. Boubendir, D. Midura, Non-overlapping domain decomposition algorithm based on modified transmission conditions for the Helmholtz equation, Computers and Mathematics with Applications (2017), http://dx.doi.org/10.1016/j.camwa.2017.07.027.