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Modeling the quantum tunneling effect for a particle with intrinsic structure in presence of external magnetic field in the Lobachevsky space

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ABSTRACT

Generalized Schrödinger equation for Cox spin zero particle is studied in presence of magnetic field on the background of Lobachevsky space. Separation of the variables is performed. An equation describing motion along the axis z turns out to be much more complicated than when describing the Cox particle in Minkowski space.

The form of the effective potential curve indicates that we have a quantum-mechanical problem of the tunneling type. The derived equation has 6 regular singular points. To physical domains $z = \pm\infty$ there correspond the singular points 0 and 1 of the derived equation. Frobenius solutions of the equation are constructed, convergence of the relevant series is examined by Poincaré–Perron method. These series are convergent in the whole physical domain $z \in (-\infty, +\infty)$. Visualization of constructed solutions and numerical study of the tunneling effect are performed.

1. Introduction

In the frames of the theory of generalized relativistic wave equations, a special model for a spinless particle was proposed by Cox [1]; some details and extensions can be seen in the Supplement. Updated treatment of this theory can be seen in recent book [2]. Such wave equations being constructed on the base of extended sets of representations of the Lorentz group, in presence of external electromagnetic fields describe after excluding additional components particles which interact nonminimally and in various ways with electromagnetic field through electromagnetic tensor. Such additional interaction terms are associated with intrinsic electromagnetic structure of the particles. In particular, the Cox electromagnetic structure in presence of external electric field may be associated with the known Darvin interaction term in nonrelativistic Schrödinger equation, this additional interaction is related to non-point-like distribution of the electric charge in the finite volume inside the particle (see, for instance, in the book [3]). In recent papers [4,5], it was studied behavior of such a particle in external magnetic and electric fields, and in spaces with non-Euclidean geometry. In particular, a generalized Schrödinger wave equation for Cox particle was derived [4].

In the present paper we examine the Cox particle in external magnetic field on the background of 3-dimensional Lobachevsky space. Influence of the curved space model becomes very significant at large distance. The problem reduces to a rather complex system of differential equations in two variables. The main attention is given to studying the equation

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describing the motion of the particle along the axis z, here we are to examine the quantum tunneling effect through an effective potential barrier.

In the special system of cylindric coordinates in the Lobachevsky space, analogue of the uniform magnetic field is determined by the relations [6] (we use dimensionless coordinates, $r/\rho \Longrightarrow z$ and so on, ρ stands for the curvature radius):

$$dS^2 = c^2dt^2 - \cosh^2 z(dr^2 + \sinh^2 r d\phi^2) - dz^2,$$
$$\sqrt{-g} = \rho^3 \ \sinh r \ \cosh^2 z, \quad A_\phi = -B\rho^2(\cosh r - 1),$$

so that

$$B_3 = F_{r\phi} = -B\rho \ \sinh r, \quad B^3 = -\frac{B}{\rho \ \sinh r \cosh^4 z}, \quad B_iB^i = B^2\cosh^{-4} z. \tag{1}$$

We start with the known form of the generalized Schrödinger equation [4] for a Cox scalar particle

$$D_t\Psi = \frac{1}{2M\rho^2}\left[\overset{\circ}{D}_1\overset{*}{D}_1 + \overset{\circ}{D}_2\frac{1}{\mathrm{sh}^2 r}\overset{*}{D}_2 + \overset{\circ}{D}_3\overset{*}{D}_3\right]\Psi,$$

where

$$D_1 = i\hbar\partial_r, \quad D_2 = i\hbar\partial_\phi + \frac{e}{c}B\rho^2(\cosh r - 1), \quad D_3 = i\hbar\partial_z,$$

$$\overset{\circ}{D}_1 = i\hbar(\partial_r + \frac{\cosh r}{\sinh r}), \quad \overset{\circ}{D}_2 = i\hbar\partial_\phi + \frac{e}{c}B\rho^2(\cosh r - 1), \quad \overset{\circ}{D}_3 = i\hbar(\partial_z + 2\frac{\sinh z}{\cosh z}),$$

$$\overset{*}{D}_1 = \frac{1}{1 + \Gamma^2B^2\cosh^{-4} z}\left[i\hbar\partial_r - \frac{\Gamma B\cosh^{-2} z}{\sinh r}(i\hbar\partial_\phi + \frac{e}{c}B\rho^2(\cosh r - 1))\right],$$

$$\overset{*}{D}_2 = \frac{1}{1 + \Gamma^2B^2\cosh^{-4} z}\left[(i\hbar\partial_\phi + \frac{e}{c}B\rho^2(\cosh r - 1)) + i\hbar\Gamma B\cosh^{-2} z \sinh r\partial_r\right],$$

$$\overset{*}{D}_3 = \frac{(D_3 + \Gamma^2B^3 \ B_3D_3)}{1 + \Gamma^2B^2\cosh^{-4} z} = i\hbar\partial_z.$$

Non-zero parameter Γ corresponds to additional structure of the Cox particle; see details in the Supplement. Below for shortness we will use notations: $B\rho^2/\hbar c = b, \ \Gamma B\cosh^{-2} z = \gamma(z)$; the sign at B (and b) relates to orientation of the magnetic field. With the use of the relations

$$\frac{1}{2M\rho^2}\overset{\circ}{D}_1 g^{11}\overset{*}{D}_1 = -\frac{\hbar^2\cosh^{-2} z}{2M\rho^2(1+\gamma^2(z))}$$
$$\times\left(\partial_r^2 + (\frac{\cosh r}{\sinh r} + i\gamma(z)b\frac{\cosh r - 1}{\sinh r})\partial_r - \frac{\gamma(z)}{\sinh r}\partial_r\partial_\phi + i\gamma(z)b\right),$$

$$\frac{1}{2M\rho^2}\overset{\circ}{D}_2 g^{22}\overset{*}{D}_2 = -\frac{\hbar^2\cosh^{-2} z}{2M\rho^2(1+\gamma^2(z))}$$
$$\times\left[\frac{1}{\sinh^2 r}[\partial_\phi - ib(\cosh r - 1)]^2 + \gamma(z)[\partial_\phi - ib(\cosh r - 1)] \ \frac{1}{\sinh r}\partial_r\right],$$

$$\frac{1}{2M\rho^2}\overset{\circ}{D}_3 g^{33}\overset{*}{D}_3 = -\frac{\hbar^2}{2M\rho^2}(\partial_z + 2\frac{\sinh z}{\cosh z})\partial_z,$$

and of the substitution for wave function

$$\Psi = e^{-iEt/\hbar}e^{im\phi}Z(z)R(r), \quad \epsilon = \frac{E}{\hbar^2/2M\rho^2}; \tag{2}$$

we derive the following equation in two variables (by physical reason – see Supplement, Eq. (57) – we make the change $\gamma \Longrightarrow i\gamma$)

$$\left(\frac{\cosh^{-2} z}{1-\gamma^2(z)}\left(\partial_r^2 + \frac{\cosh r}{\sinh r}\partial_r - \frac{[m - b(\cosh r - 1)]^2}{\sinh^2 r} + b\gamma(z)\right)\right.$$
$$\left. + \epsilon + (\partial_z + 2\frac{\sinh z}{\cosh z})\partial_z\right)R(r)Z(z) = 0. \tag{3}$$