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Fractal dimensions of random attractors for stochastic Benjamin–Bona–Mahony equation on unbounded domains☆

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A B S T R A C T

This paper considers random attractor and its fractal dimension for Benjamin–Bona–Mahony equation driven by additive white noise on unbounded domains . Firstly, we investigate the existence of random attractor for the random dynamical system defined on an unbounded domain. Secondly, we present criterion for estimating an upper bound of the fractal dimension of a random invariant set of a random dynamical system on a separable Banach space. Finally, we apply expectations of some random variables and these conditions to prove the finiteness of fractal dimension of the random attractors for stochastic Benjamin–Bona–Mahony equation driven by additive white noise.

1. Introduction

In this paper, we consider the existence of random attractor and fractal dimension of random attractor for the following non-autonomous stochastic Benjamin–Bona–Mahony (BBM) equation with additive white noise on unbounded domains. Let $U = U_0 \times \mathbb{R}$ where U_0 is a bounded domain in $\mathbb{R}^2$, $x = (x_1, x_2, x_3) \in U$. Consider the following BBM equation on U:

$$du - d(\Delta u) - \gamma \Delta u dt + \nabla \cdot \vec{F}(u)dt = g(x,t)dt + hdW, \quad x \in U, \quad t > 0, \tag{1.1}$$

where γ is a positive constant, the non-linear term $\vec{F}(u)$ is a smooth vector function, both $h(x)$ and the forcing term $g(x,t)$ are given functions, $g \in L^2_{loc}(\mathbb{R}, L^2(U))$. $W(t)$ is a two-side real-valued Wiener process on the probability space $(\Omega, \mathcal{F}, \mathbb{P})$ where $\Omega = \{\omega \in C(\mathbb{R}, \mathbb{R}) : \omega(0) = 0\}$. The Borel σ-algebra $\mathcal{F}$ on Ω is generated by the compact open topology, and $\mathbb{P}$ is the corresponding Wiener measure on $\mathcal{F}$.

We all know that the random attractor is used to describe the long-term behavior of random dynamical systems. Now, the existence of the attractors theory for many deterministic systems and autonomous random dynamical systems has been studied, see [1–3]. However, when the stochastic dynamical system is non-autonomous, especially, for PDEs defined on unbounded domains, the existence of the random attractor is not well understood. Fortunately, authors in [4,5] put forward the concept of $\mathcal{D}$-pullback attractor describing the long-time behavior of non-autonomous random dynamical system, and a few authors have investigated the existence of $\mathcal{D}$-pullback attractor for some partial differential equations, see [6–11]. And until now, there have several useful methods to estimate the upper bound of the Hausdorff and fractal dimensions of

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a random attractor, see [12–16]. In the case of bounded domains, the existence and fractal dimension of random attractors for stochastic PDEs have been studied by many authors, see [12,13]. However, for PDEs defined on unbounded domains, the existence of such attractors is not well understood. In this paper, we study Eq. (1.1) which is a non-autonomous equation driven by additive white noise. We will consider the random attractor for stochastic system (1.1) from two respect:

First, we prove the existence of random attractor of the stochastic BBM system (1.1). Notice that the domain U is unbounded, then the Sobolev embedding on U is invalid. This introduces a major obstacle for proving the existence of random attractors. We transform (1.1) into a deterministic one with only one random parametric. Author in [17] advanced the tail-estimates approach to solve that problem, and this method is applicable, see [18,19]. Wang in [2] proved the existence of a compact random attractor for the stochastic Benjamin–Bona–Mahony equation defined on an unbounded domain. In addition, forcing term $g(t)$ is time-dependent, which is a new difficult. Wang in [20] studied the existence of a pullback global attractor for the non-autonomous reaction–diffusion equations on $\mathbb{R}^n$. Authors showed the existence of a pullback attractor for the non-autonomous Benjamin–Bona–Mahony equation in unbounded domains without noise, see [1]. Motivated by the ideas in [1,2,17–20], we use the techniques of uniform estimates on the tails of solution to prove the existence of random attractors for stochastic Benjamin–Bona–Mahony equation driven by white noise.

Second, this paper presents the upper bound of fractal dimension of random attractor without the differentiability of random dynamical system by the boundedness of expectation of these random variables. Debussche in [14] estimated the Hausdorff dimension of a random invariant set of a random map by using a random squeezing property and Lyapunov exponents. Motivated by the method of [14], Langa studied the fractal dimension of a random invariant set, see [16]. Similarly, Wang and Tang provided a method to figure out the fractal dimension of random attractor, see [15], but which are requiring the differentiability of random dynamical system or requiring that the Lipschitz constant of system and the "contractio" coefficient of the infinite dimensional part of system be independent of the sample points. Debussche overcame this difficulty and provided some conditions to estimate the Hausdorff dimension of a random attractor, see [21]. Based on [21], concretely, by establishing a criteria for bounding the fractal dimension of a random invariant set under a non-autonomous random dynamical system and giving some special conditions, Zhou obtained an upper bound of fractal dimension of random attractor for stochastic damped wave equation with additive white noise [13]. Motivated by the ideas of [12,13,21]. This paper gives some conditions for bounding the fractal dimension of a random invariant set for a random dynamical system which does not require the differentiability of random dynamical system. We establish some random variables and just prove the boundedness of expectation of these random variables. We apply these conditions to obtain an upper bound of fractal dimension of random attractor for (1.1).

In the sequel, we adopt the following notations. We denote by $\|\cdot\|$ and $(\cdot,\cdot)$ the norm and the inner product of $L^2(U)$, respectively. $\|\cdot\|_X$ denotes the norm in Banach space X, and $\|\cdot\|_p$ denotes the norm in $L^p(U)$. Letters c and $c_i (i = 1, 2, \ldots)$ are positive constants which may change their values from line to line or even in the same line. The following inequalities will be frequently used in this paper.

$$\|u\|_\infty \le \beta_0 \|u\|_{H^2(U)}, \quad \forall u \in H^2(U), \tag{1.2}$$

$$\|\nabla u\|^2 \ge \lambda \|u\|^2, \quad \forall u \in H_0^1(U), \tag{1.3}$$

where β_0 and λ are positive constants.

Throughout this article, for simplicity, we identify "$a.e.\omega \in \Omega$" with "$\omega \in \Omega$".

2. Mathematical setting

Consider the initial boundary value problem of (1.1)

$$\begin{cases} du - d(\Delta u) - \gamma \Delta u dt + \nabla \cdot \vec{F}(u)dt = g(x,t)dt + h(x)dW, & x \in U, \quad t > 0, \\ u(x,t)|_{\partial U} = 0, \quad x \in U, \quad t \ge \tau, \\ u(x,\tau) = u_\tau(x), \quad x \in U, \end{cases} \tag{2.1}$$

where the non-linear term $\nabla \vec{F}(s)$ is a smooth vector function given by $\vec{F} = (F_1(s), F_2(s), F_3(s))$, which satisfies

$$F_i(0) = 0, \quad |F_i'(s)| \le \beta_1 + \beta_2 |s|, \quad i = 1, 2, 3. \; s \in \mathbb{R}. \tag{2.2}$$

Denote by

$$G_k(s) = \int_0^s F_k(t)dt \quad and \quad \vec{G}(s) = (G_1(s), G_2(s), G_3(s)). \tag{2.3}$$

Then from (2.2)–(2.3), we have

$$|F_i| \le \beta_1 |s| + \beta_2 |s|^2 \quad and \quad |G_i(s)| \le \beta_1 |s|^2 + \beta_2 |s|^3, \quad i = 1, 2, 3, \tag{2.4}$$

where $\beta_1, \ \beta_2$ are positive constants. And denote by

$$\delta = \min\{\gamma, \frac{\gamma\lambda}{4}\}, \quad \beta = 4\beta_0\beta_2 \|h\|_{H^1}, \tag{2.5}$$

where β_0, λ are positive numbers in (1.2) and (1.3).

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