ARTICLE IN PRESS

Computers and Mathematics with Applications ▮ (▮▮▮▮) ▮▮▮–▮▮▮

Contents lists available at ScienceDirect

Computers and Mathematics with Applications

journal homepage: www.elsevier.com/locate/camwa

Global asymptotic stability of steady states in a chemotaxis-growth system with singular sensitivity

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ARTICLE INFO

Article history:
Received 29 March 2017
Received in revised form 19 October 2017
Accepted 26 November 2017
Available online xxxx

Keywords:
Asymptotic stability
Chemotaxis
Singular sensitivity

ABSTRACT

This paper deals with a fully parabolic chemotaxis-growth system with singular sensitivity

$$\begin{cases} u_t = \Delta u - \chi\nabla\cdot(u\nabla\ln v) + ru - \mu u^2, & (x,t)\in\Omega\times(0,\infty),\\ v_t = \Delta v - v + u, & (x,t)\in\Omega\times(0,\infty), \end{cases}$$

under homogeneous Neumann boundary conditions in a smooth bounded domain $\Omega\subset\mathbb{R}^2$, where the parameters $\chi,\mu>0$ and $r\in\mathbb{R}$. Global existence and boundedness of solutions to the above system were established under some suitable conditions by Zhao and Zheng (2017). The main aim of this paper is further to show the large time behavior of global solutions which cannot be derived in the previous work.

1. Introduction

In this paper, we consider the following fully parabolic chemotaxis-growth system with singular sensitivity (see [1]):

$$\begin{cases} u_t = \Delta u - \chi\nabla\cdot(u\nabla\ln v) + ru - \mu u^2, & (x,t)\in\Omega\times(0,\infty),\\ v_t = \Delta v - v + u, & (x,t)\in\Omega\times(0,\infty),\\ \dfrac{\partial u}{\partial\nu} = \dfrac{\partial v}{\partial\nu} = 0, & (x,t)\in\Omega\times(0,\infty),\\ u(x,0)=u_0(x),\ v(x,0)=v_0(x), & x\in\Omega, \end{cases} \tag{1.1}$$

where $\chi,\mu>0$ and $r\in\mathbb{R}$, $\Omega\subset\mathbb{R}^2$ is a bounded domain with smooth boundary $\partial\Omega$, $\frac{\partial}{\partial\nu}$ denotes the differentiation with respect to the outward normal derivative on $\partial\Omega$, and the initial data

$$\begin{cases} u_0\in C^0(\overline{\Omega}),\ u_0(x)\geq 0 \text{ and } u_0(x)\not\equiv 0,\ x\in\overline{\Omega}, \quad \text{and}\\ v_0\in W^{1,p}(\Omega) \quad \text{for some } p>2,\ v_0(x)>0,\ x\in\overline{\Omega}. \end{cases} \tag{1.2}$$

Chemotaxis is an oriented movement of biological cells or organisms in response to a chemical gradient. The pioneering works of chemotaxis models were introduced by Patlak [2] in 1953 and Keller and Segel [3] in 1970, and we refer the readers to the survey [4–7] where a comprehensive information of further examples illustrating the outstanding biological relevance of chemotaxis can be found. The sensitivity function $\frac{1}{v}$ in system (1.1) is considered by the Weber–Fechner laws describing

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https://doi.org/10.1016/j.camwa.2017.11.032
0898-1221/

Please cite this article in press as: P. Zheng, et al., Global asymptotic stability of steady states in a chemotaxis-growth system with singular sensitivity, Computers and Mathematics with Applications (2017), https://doi.org/10.1016/j.camwa.2017.11.032.

the cells response to the chemical signal [8]. In order to understand system (1.1), let us mention some previous contributions in this direction. In recent years, the following initial boundary value problems have been studied by many authors:

$$\begin{cases} u_t = \Delta u - \nabla\cdot(u\chi(v)\nabla v), & (x,t)\in\Omega\times(0,\infty),\\ v_t = \Delta v - v + u, & (x,t)\in\Omega\times(0,\infty),\\ \dfrac{\partial u}{\partial \nu} = \dfrac{\partial v}{\partial \nu} = 0, & (x,t)\in\partial\Omega\times(0,\infty),\\ u(x,0)=u_0(x),\ v(x,0)=v_0(x), & x\in\Omega, \end{cases} \tag{1.3}$$

where $\Omega\subset\mathbb{R}^n$, $n\geq 1$ is a bounded domain with smooth boundary $\partial\Omega$. Winkler [9] proved that system (1.3) has a unique globally bounded classical solution provided that $0<\chi(v)\leq\frac{\chi_0}{(1+\alpha v)^k}$ with some $\alpha>0$ and $k>1$ for any $\chi_0>0$. Recently, Fujie and Yokota [10] extended this result of [9] to the singular case $0<\chi(v)\leq\frac{\chi_0}{v^k}$ for some $\chi_0>0$ and $k>1$. Moreover, Zhang and Li [11] generalized the result of [9] to a two-species chemotaxis system. Furthermore, when $n\geq 2$ and $\chi(v)=\frac{\chi_0}{v}$ in [12], the global existence of classical solution to (1.3) is proved if $\chi_0<\sqrt{\frac{2}{n}}$, while $\chi_0<\sqrt{\frac{n+2}{3n-4}}$, then the global existence of weak solution is established. Fujie [13] obtained the boundedness result in (1.3) with $\chi(v)=\frac{\chi_0}{v}$ under the cases $0<\chi_0<\sqrt{\frac{2}{n}}$ and $n\geq 2$. In the radially symmetric setting, Stinner and Winkler [14] constructed certain weak solutions of (1.3) with $\chi(v)=\frac{\chi_0}{v}$ and $\chi_0<\sqrt{\frac{n}{n-2}}$ provided that $n\geq 3$. Invoking additional dampening kinetic terms, Manásevich et al. [15] proved global existence and boundedness in a related system for any $\chi_0>0$. On the other hand, when chemicals diffuse much faster than cells move, the corresponding simplified parabolic–elliptic chemotaxis systems in (1.3) have also been studied by some authors (see [16–19]). Moreover, Fujie et al. [20] studied the global boundedness for the simplified parabolic–elliptic chemotaxis system with $\chi(v)=\frac{\chi_0}{v}$ invoking additional dampening kinetic terms. Moreover, Cao et al. [21] proved that the global solution converges asymptotically to the constant equilibrium. Recently under the standard logistic growth case, Zhao and Zheng [1] extended the boundedness result of [20] to the fully parabolic system with the singular sensitivity $\frac{1}{v}$. However, the large time behavior of global solutions to (1.1) remains open in the previous works.

To the best of our knowledge, only few results on the large time behavior were studied in (1.1). Our main purpose in this paper is to further investigate the global asymptotic stability of global solutions to system (1.1).

Our main result in this paper is stated as follows.

Theorem 1.1. *Let $\Omega\subset\mathbb{R}^2$ be a bounded smooth domain, $\chi>0$, $\mu>\max\{\frac{1}{2},\frac{\chi^2 r}{2\delta_0^2}\}$ and*

$$r>\begin{cases}\dfrac{\chi^2}{4}, & 0<\chi\leq 2,\\ \chi-1, & \chi>2,\end{cases} \tag{1.4}$$

where δ_0 is given by Lemma 2.2. Assume that the initial data (u_0,v_0) satisfies (1.2). Then the global bounded solution (u,v) of (1.1) exponentially converges to the steady state, i.e., there exist positive constants C and λ such that

$$\|u(\cdot,t)-\frac{r}{\mu}\|_{L^\infty(\Omega)}+\|v(\cdot,t)-\frac{r}{\mu}\|_{L^\infty(\Omega)}\leq Ce^{-\lambda t}\quad \text{for all}\ \ t>0.$$

Remark 1.1. Compared with the previous results in [22], we further show the exact convergence rate of global solutions.

2. Regularities of bounded solutions

In this section, we provide more strong regularity properties for any such bounded solution than those shown in [1], which are needed to achieve our desired rates of convergence in L^∞-norm. To do this, we shall collect the L^∞-boundedness of solutions for system (1.1) as follows.

Lemma 2.1. *Let $\Omega\subset\mathbb{R}^2$ be a bounded smooth domain, $\chi>0$, $\mu>0$ and r satisfies (1.4). Then for any initial data satisfying (1.2), system (1.1) possesses a unique global nonnegative classical solution, which is uniformly bounded in $\Omega\times(0,\infty)$ in the sense that there exists a positive constant C such that*

$$\|u(\cdot,t)\|_{L^\infty(\Omega)}+\|v(\cdot,t)\|_{L^\infty(\Omega)}\leq C\quad \text{for all}\ \ t>0. \tag{2.1}$$

Proof. The boundedness result of Lemma 2.1 directly comes from Theorem 1 in [1]. Thus we delete the details. □

Next we only show the following uniform-in-time lower bound of v in Ω with r satisfying (1.4), which is proved in Lemma 3.3 of [1].

Lemma 2.2. *Suppose that the conditions of Lemma 2.1 hold. Then there exists $\delta_0>0$ such that $v(x,t)\geq\delta_0$ for all $(x,t)\in\Omega\times(0,\infty)$.*