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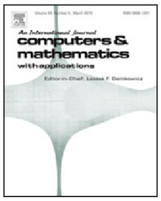

A meshless average source boundary node method for steady-state heat conduction in general anisotropic media

Yao-Ming Zhang [a,*], Fang-Ling Sun [b], Wen-Zhen Qu [a], Yan Gu [c], Der-Liang Young [d]

[a] *Institute of Applied Mathematics, Shandong University of Technology, Zibo 255049, China*
[b] *Department of Mechanics, School of Aerospace Engineering, Beijing Institute of Technology, Beijing 100081, China*
[c] *College of Mathematics, Qingdao University, Qingdao 266071, China*
[d] *Department of Civil Engineering and Hydrotech Research Institute, National Taiwan University, Taipei 10617, Taiwan*

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ABSTRACT

The average source boundary node method (ASBNM) is a recent boundary-type meshless method, which uses only the boundary nodes in the solution procedure without involving any element or integration notion, that is truly meshless and easy to implement. This paper documents the first attempt to extend the ASBNM for solving the steady-state heat conduction problems in general anisotropic media. Noteworthily, for boundary-type meshless/meshfree methods which depend on the boundary integral equations, whatever their forms are, a key but difficult issue is to accurately and efficiently determine the diagonal coefficients of influence matrices. In this study, we develop a new scheme to evaluate the diagonal coefficients via the pure boundary node implementation based on coupling a new regularized boundary integral equation with direct unknowns of considered problems and the average source technique (AST). Seven two- and three-dimensional benchmark examples are tested in comparison with some existing methods. Numerical results demonstrate that the present ASBNM is superior in the light of overall accuracy, efficiency, stability and convergence rates, especially for the solution of the boundary quantities.

1. Introduction

Nowadays new type materials exhibiting characteristics of anisotropy are widely used in the industries thanks to their attractive properties. Therefore, heat conduction in these materials has numerous important applications in various branches of science and engineering [1–3] and, hence, the numerical simulation of heat conduction in this type of material is of great importance.

In the past several decades, meshless/meshfree numerical method [4–10] has drawn growing interests from researchers and appears to have become an effective and significant alternative to dominant mesh-based numerical methods such as the finite element method (FEM) [11] and the boundary element method (BEM) [12–14] for some realistic engineering applications thanks to their easy model setup characteristic that neither medium domain nor its interface meshing is required. In general, these methods can be sorted into the domain-type technique and the boundary-type technique. Compared with the domain-type meshless methods, the boundary-type meshless approaches could be more attractive

* Correspondence to: Institute of Applied Mathematics, Shandong University of Technology, Zibo 255049, Shandong Province, China.
E-mail address: zym@sdut.edu.cn (Y.-M. Zhang).

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and competitive for researchers to use, since they preserve the dimensionality and simple modeling of exterior problems advantages of boundary numerical methods, such as the BEM.

In recent years, a number of boundary-type meshless methods [6–10,15–32] have been developed and gained great success in solving a broad range of science and engineering problems. Some of them, such as the method of fundamental solution (MFS) [3,15,16], the boundary collocation method (BCM) [17], the boundary knot method (BKM) [18,19] is founded on the notion of the method of fundamental solution (MFS). Hence, although mathematically simple, easy-to program and sometime even very accurate, these methods generally lead to the ill-conditioned discretization system. Some other approaches, such as the modified MFS (MMFS) [20], the regularized meshless method (RMM) [21,22], the boundary distributed source method (BDSM) [23,24], and the singular boundary method (SBM) [25–27], although called as the modified/improved MFS, really depend on boundary integral equation. Noteworthily, for any boundary-type meshless/meshfree method which depends on the boundary integral equations, whatever its forms are, a crucial but difficult issue is to efficiently and accurately determine the diagonal coefficients from both the fundamental solution and its derivative of considered problems. The MMFS and the BDSM determine the diagonal coefficients of influence matrices by the computation of some particular integrals. The RMM uses double-layer potential to express the solution of the problem to easily evaluate the diagonal coefficients from the normal derivative of the fundamental solution, but the bewildering hypersingularity leads to its low accuracy. The SBM firstly determines the diagonal coefficients from the normal derivative of the fundamental solution via the null-field integral equations, and then an inverse interpolation technique (a known solution) is used to evaluate the diagonal coefficients from the fundamental solution [25–27]. Therefore, as shown in Ref. [23], this approach requires to solve considered problems twice. Recently, a new improvement of the SBM was given in Ref. [28], which proposed a new algorithm to directly compute the diagonal coefficients of influence matrices employing the computational strategies of the singular integrals in BEM. And thus this algorithm is accurate but inefficient due to its requirement of the complicated mathematical deduction based on the cell/segment geometry. Šarler and his co-workers presented the method of regularized sources (MRS) for Stokes flow problems [29,30] and the non-singular MFS for anisotropic elasticity [31]. The MRS and the non-singular MFS both belong to MFS-type approach, which avoid the disadvantage of artificial boundary in the conventional MFS.

Very recently, Zhang and his co-workers have developed a new boundary-type meshless method, called the average source boundary node method(ASBNM) [32–34]. The method established a 'completely' regularized boundary integral equation (CRBIE) of potential problems, by which all diagonal coefficients of influence matrices can be determined accurately and efficiently. Another novelty of the ASBNM lies in introducing an average source technique (AST) into the BEM. The AST seems very simple, but plays a crucial role in establishing the meshless ASBNM. On the one hand the use of the AST makes the distributed source on a segment reduced to the point source in the limit sense and thus the boundary integrals are unnecessary. On the other hand, very importantly, the AST can also directly compute the diagonal coefficients with the strong singularity of influence matrix or improve the accuracy of the diagonal coefficients computation provided by the regularized boundary integral equation (RBIE) via a limit procedure [34].

In this study, we make the first attempt to extend the ASBNM for the solution of the steady-state heat conduction problems in general anisotropic media. An accurate and efficient algorithm to evaluate the diagonal coefficients of influence matrices, an essentially crucial issue for any boundary-type meshless methods, is proposed via a pure boundary node implementation based on a new CRBIE of considered problems established here in conjunction with the AST. It is worth noting that although the vein of the present algorithm is similar to that of the previous ASBNM in potential problems, it is still an important new development due to the CRBIE together with the AST being problem-dependent. To establish the CRBIE with direct unknowns, the usual Green' second identity for the harmonic operator is firstly extended to a general form for the complete second order differential operator with variable coefficients in 2D or 3D space. And then several integral identities for the fundamental solution are established by such a new Green' identity. Based on these, a CRBIE is developed via a limit procedure [35].

This paper is organized as follows. The problem under consideration is described mathematically in Section 2. Next, in Section 3, we derive the CRBIE with direct unknowns and the ASBNM. Some numerical experiments are presented in Section 4 to verify the proposed method. Finally, the paper ends with some conclusions and remarks in Section 5.

2. Model formulation

Consider the steady heat conduction in an anisotropic medium which is in a bounded domain $\Omega \subset R^d$, with $d = 2, 3$ being the dimensionality of the space. In the absence of heat sources, the temperature distribution u in Ω satisfies the following differential equation

$$\nabla \cdot (\boldsymbol{K} \nabla u(\boldsymbol{x})) = k_{ij} \partial_i \partial_j u(\boldsymbol{x}) = 0,\ (i, j = 1, \ldots, d),\ \boldsymbol{x} \in \Omega \tag{1}$$

where $\partial_i \equiv \partial/\partial x_i$, $\boldsymbol{K} = [k_{ij}]_{1 \le i,j \le d}$ is the thermal conductivity tensor which is invariant in space and is assumed to be symmetric and positive-definite so that the partial differential Eq. (1) is elliptic, and herein the summation convention is used with respect to the repeated indices. These assumptions will be applied also in what follows unless specified others.

Let $\boldsymbol{n} = (n_1, \ldots, n_d)$ be the unit outward normal vector at boundary point $\boldsymbol{x} \in \Gamma = \partial\Omega$. The normal heat flux $q(\boldsymbol{x})$ at a point $\boldsymbol{x} \in \Gamma$ is defined as [1]

$$q(\boldsymbol{x}) = \boldsymbol{n}(\boldsymbol{x}) \cdot (\boldsymbol{K} \nabla u(\boldsymbol{x})) = \boldsymbol{n} \cdot \boldsymbol{F} = \nabla u \cdot \boldsymbol{m},\ \ \boldsymbol{x} \in \Gamma \tag{2}$$

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