Computers and Mathematics with Applications ▮ (▮▮▮▮) ▮▮▮–▮▮▮

Contents lists available at ScienceDirect

Computers and Mathematics with Applications

journal homepage: www.elsevier.com/locate/camwa

A study on the 3D position estimation of ventricular borders extracted from 2D echocardiography data

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ARTICLE INFO

Article history:
Received 27 June 2017
Received in revised form 26 September 2017
Accepted 28 October 2017
Available online xxxx

Keywords:
Echocardiography
Left ventricle
3D reconstruction
Mathematical model
Quadratically constrained quadratic program

ABSTRACT

We study on the reconstruction of 3D left ventricle(LV) using only 2D echocardiography data and information on apical long-axis views. Especially, this paper focuses on determining the 3D position of LV contours extracted from 2D echocardiography images. First we mathematically model the relationship between LV contours on the apical views and their corresponding 3D positions. The relationship is expressed as a linear equation in which the right-hand side is the measured data consisting of all the LV contour points on each view and the coefficient matrix is an unknown matrix that transforms the unknown 3D positions into contour points on their related apical view, with distance and orthogonality conditions on the coefficient matrix and the 3D positions. Next we consider a non-convex constrained minimization problem to determine the coefficient matrix and the 3D positions. To solve this minimization problem, we adopt two block coordinate descent method with a solver in *OPTI* for quadratically constrained quadratic program. For validating the proposed method, some numerical experiments are performed with synthetic data. The experimental results show that the proposed model is promising and available for real echocardiography data.

1. Introduction

Ultrasound imaging system has become a widely used diagnostic tool because of its various advantages such as non-invasiveness, safety, portability, real-time imaging and inexpensive cost, compared with other medical imaging modalities including X-ray, CT, and MRI [1–3]. Especially, for cardiac ultrasound examinations, it provides anatomical information on cardiac motion as well as functional information by computing quantitative indices such as LV hypertrophy, stroke volume, ejection fraction, cardiac output, and so on. The quantitative indices provide functional information and are utilized to evaluate heart disease clinically [4–6].

In company with the emergence of real-time 3D echocardiography(RT3DE), the demands for analysis tools to assess the LV function using RT3DE are steadily increasing [7,8]. Nevertheless, most of analysis tools are still based on the measurements in 2D slices because they are time- and cost-efficient and available in clinical practice.

In this paper, we propose a mathematical reconstruction model of 3D LV using only 2D echocardiography data without using a 3D ultrasound imaging scanner. For reconstructing the 3D LV shape, LV contour is extracted from 2D echocardiography data in each apical view and all the 2D LV contours are covered together by a smooth surface in 3D-space. The problem to be solved is how to determine the 3D position of LV contours extracted in each view. We compute the 3D coordinates of

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https://doi.org/10.1016/j.camwa.2017.10.025
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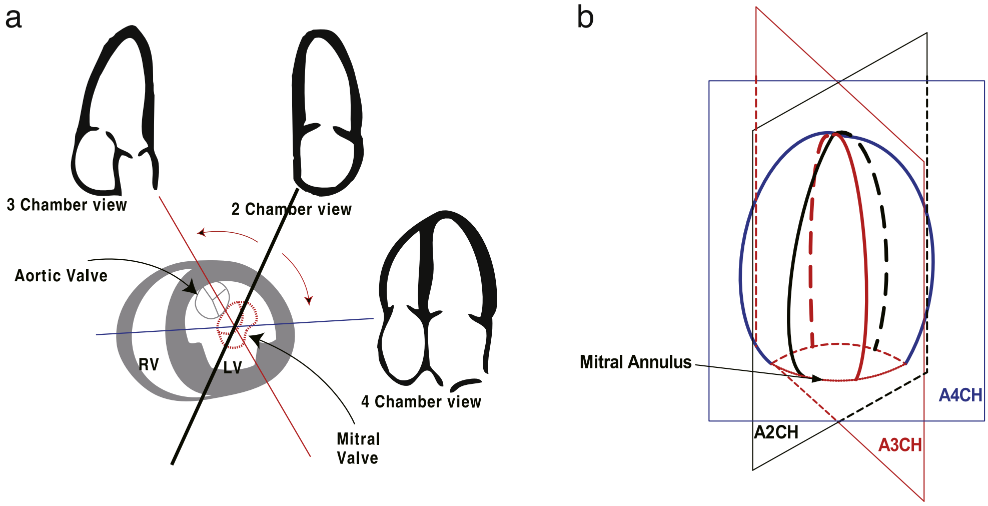

Fig. 1. Description on apical 4-chamber, 2-chamber and 3-chamber views of cardiac images depending on scanning probe. They are obtained by rotating the scanning probe clockwise and counterclockwise approximately 60°, respectively, in order.

the LV contour points using the fact that the angles between apical long-axis 4, 2 and 3-chamber views are approximately 60° toward each other [6], as shown in Fig. 1. However, we notice that the angles cannot be exactly 60° in actual situation.

In the proposed model, we express the relationship between the extracted 2D LV contours and the corresponding 3D positions as a linear equation of the form $\mathbf{Ax} = \mathbf{b}$, where $\mathbf{b}$ is measured data consisting of all the LV contour points on each view, $\mathbf{x}$ is corresponding unknown 3D positions, and $\mathbf{A}$ is an unknown matrix that transforms the 3D positions into contour points on their related apical view, with the conditions that circumferential length along mitral annulus is in the given range and $\mathbf{A}$ satisfies orthogonality constraints, i.e., $\mathbf{A}^T\mathbf{A} = \mathbf{I}$ and $\mathbf{A}\mathbf{A}^T = \mathbf{I}$.

In order to determine $\mathbf{A}$ and $\mathbf{x}$, we consider a non-convex constrained minimization problem. The given minimization problem is a quadratically constrained quadratic program(QCQP) [9] in terms of $\mathbf{x}$ if $\mathbf{A}$ is fixed and is also a QCQP in terms of $\mathbf{A}$ if $\mathbf{x}$ is fixed. To solve this minimization problem, we adapt two block coordinate descent method with a solver in *OPTI* [10] for QCQP. In other words, we alternately minimize the given model with respect to $\mathbf{x}$ and then with respect to $\mathbf{A}$ by using the QCQP solver.

We test the proposed model on synthetic data of two different shapes which are an ellipsoid shape and a peanut shape. Numerical results show that the proposed model is promising.

There have been several approaches for constructing 3D volume from 2D ultrasound images: sweeping techniques acquiring a series of parallel slices using a motor attached to a probe, image acquisition by 3D probes consisting of 2D arrays, freehand system allowing image acquisition with unconstrained probe movement, and so on. In those approaches, the 3D position and orientation of 2D image planes are spatially pre-determined or tracked using a sensor attached to a probe [11]. To the best of our knowledge, there has been no study on mathematical modeling for the 3D LV shape reconstruction from several 2D LV contours and thus this work is the first attempt.

2. Reconstruction model

In this section, we formulate a mathematical model to reconstruct the 3D LV surface from 2D ultrasound images acquired independently in each apical view. Before developing the mathematical reconstruction model to estimate the 3D positions corresponding to the 2D LV contours, we define some notations. Let Ω be an imaging domain, $\mathbf{t} = (x_o, y_o)$ a vector translating the origin to the inside region of LV contours in Ω, θ the angle to rotate counterclockwise around the translated origin and φ the angle to rotate around y-axis in 2D echocardiography data to represent the LV contours in a 3D spatial domain, respectively (see Fig. 2). And we denote the LV contour in an ultrasound image by a parametric contour $\mathcal{C} = \{\mathbf{r}(s) = (x(s), y(s)) \ : \ 0 \le s \le 1\}$ that can be identified as its n contour points $\mathbf{r}_1 = \mathbf{r}(s_1), \dots, \mathbf{r}_n = \mathbf{r}(s_n)$, where $0 = s_1 < s_2 < \cdots < s_n = 1$. For the ith point $\mathbf{r}_i = (x_i, y_i)$, we denote the extended 3D coordinate and the geometrically transformed 3D position by $\tilde{\mathbf{r}}_i = (x_i, y_i, 0)$ and $\mathbf{R}_i = (X_i, Y_i, Z_i)$, respectively. The identity matrix is denoted by I and the matrix of zero entries is denoted by $\mathbf{O}_{n\times n}$.

If the coordinates are regarded as column vectors, the 3D position $\mathbf{R}_i$ can be written as

$$\mathbf{R}_i = \Psi(\tilde{\mathbf{r}}_i - \tilde{\mathbf{t}}), \tag{1}$$