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Discrete kinetic theory for 2D modeling of a moving crowd: Application to the evacuation of a non-connected bounded domain

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ABSTRACT

This paper concerns the mathematical modeling of the motion of a crowd in a non connected bounded domain, based on kinetic and stochastic game theories. The proposed model is a mesoscopic probabilistic approach that retains features obtained from both micro- and macro-scale representations; pedestrian interactions with various obstacles being managed from a probabilistic perspective. A proof of the existence and uniqueness of the proposed mathematical model's solution is given for large times. A numerical resolution scheme based on the splitting method is implemented and then applied to crowd evacuation in a non connected bounded domain with one rectangular obstacle. The evacuation time of the room is then calculated by our technique, according to the dimensions and position of a square-shaped obstacle, and finally compared to the time obtained by a deterministic approach by means of randomly varying some of its parameters.

1. Introduction

The dynamic modeling of crowd motion has recently aroused a great interest in the scientific community and is used in numerous applications, such as engineering and social science [1]. It has become increasingly important to avoid or control panic situations and to ensure the safety of people in congested areas.

Mathematical representations of crowd motion from the microscopic to macroscopic scale have been an active field of study for the last two decades, with a rich scientific literature [2–6]. The aim of this paper is not to present an exhaustive list of references. Only some of the most frequently used will be mentioned: the microscopic approach based on the social forces model, proposed by Helbing [4,5], where the movement of the crowd is characterized by the position and velocity of each individual, and the macroscopic models, given by Hughes [6], that consider the crowd as a fluid.

Recently, an intermediate mesoscopic representation based on the kinetic approach appeared and its application to crowd representation gave promising results for the description of pedestrians' strategy. Very few references of the mesoscopic representation can be found in the literature [7–11].

The kinetic theory is a mathematical description of a volume of material containing a large number of particles interacting with each other, for example, a volume of gas particles [12]. This approach allows us to connect both the macroscopic

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and microscopic properties. Monte Carlo particle methods have a relevant role in the numerical resolution of kinetic equations [13,14]. Moreover, this theory has been applied in many areas, namely, modeling of vehicular traffic [15,16], and crowd dynamics [7–11,17,18], which is the subject of this study.

The modeling of a crowd by a kinetic approach started with Bellomo's and Bellouquid's work [9], in which the set of main governing equations are introduced. In this approach, the crowd is seen as a complex system in which the interactions between people (particles) are managed from a probabilistic point of view and the microscopic state of each pedestrian (particle) is characterized by his/her position and speed. In addition, the general form of the system is represented by a distribution function in a microscopic state and the dynamic of this distribution function is given by the study of particles' balance in a unit volume element of the phase plane. Then Bellomo et al. [8] developed this approach and handled the movement of a crowd moving in different directions, in an unbounded domain and where the objective of each particle is to reach a fixed target. Afterward, Agnelli et al. [7] studied the case of people moving in a connected bounded domain, without obstacles. Then Bellomo and Gibelli treated the density–velocity diagram in steady flow conditions and studied some collective emerging behaviors that are experimentally observed, namely the self-organized behaviors leading to the creation of lanes in streets and the increasing of evacuation time in stressful conditions [17].

Most of the studies that were previously mentioned, concerned connected areas [7–9,17], while the question of non connected areas is still open. In this paper, the kinetic theory applied to crowd dynamics is extended to its motion in a non-connected bounded domain, with the presence of fixed obstacles. To model interactions, it is assumed that pedestrians can change their direction for various reasons, such as: the wish to reach a target, the avoidance of the edges of the domain and/or fixed obstacles in the field. In a future step, pedestrians will be considered as "active particles" by taking into account their heterogeneity and their capacity to develop a strategy of displacement.

This paper is organized as follows: Section 2 provides the mathematical model for the crowd evacuation in an area including walls and obstacles. Then, probabilistic tools are used to describe pedestrian–pedestrian interactions as well as pedestrians interactions along with the geometry of the area. Section 3 presents a mathematical framework to obtain proofs of the existence and of the uniqueness of the proposed model's solution. Section 4 is devoted to numerical simulations to check the ability of the proposed model to describe the main features of the pedestrian dynamics, particularly the avoidance of fixed obstacles on their walk toward the exit. The influence of the position of a fixed square obstacle in the vicinity of the exit is finally studied with respect to the evacuation time for a group of 50 persons.

2. Position of the problem under study. Mathematical modeling

Let us consider a system composed of N particles (the pedestrians) distributed randomly in a two-dimensional bounded domain $\Omega \subset \mathbb{R}^2$.

This group of N pedestrians present in the room at initial time t_0, wish to evacuate the room by the exit of size S. At initial time $t = t_0$, pedestrians are distributed within a disk $\mathcal{D}_0$ of radius r and of center $M_0(x_0, y_0)$. The initial global density is then: $\rho_0 = \frac{N}{\pi r^2}$ (ped/m^2).

Kinetic type equations derivation requires a detailed analysis of the interactions at a micro-scale, namely at the pedestrian scale related to the statistical representation of the overall system; this requires a suitable probability distribution over the micro-state.

This particle distribution function is given by: $f = f(t, \mathbf{x}, \mathbf{v})$ for all $t \geq t_0, \mathbf{x} \in \Omega, \mathbf{v} \in D_\mathbf{v}$, where $D_\mathbf{v}$ represents the domain of velocities.

If $f(t, \mathbf{x}, \mathbf{v})$ is locally integrable in $\mathbf{x}$, then $f(t, \mathbf{x}, \mathbf{v})d\mathbf{x}d\mathbf{v}$ represents the number of individuals, located at time t in an infinitesimal rectangle $[x, x + dx] \times [y, y + dy]$ with the velocity belonging to $[v_x, v_x + dv_x] \times [v_y, v_y + dv_y]$, where: $\mathbf{x} = (x, y)$ and $\mathbf{v} = \left(v_x, v_y\right)$.

If $f(t, \mathbf{x}, \mathbf{v})$ is locally integrable in $\mathbf{v}$, the local density (the number of people per square meter) at the point $\mathbf{x}$ and time t can be introduced:

$$\rho\,(t, \mathbf{x}) = \int_{D_\mathbf{v}} f(t, \mathbf{x}, \mathbf{v})d\mathbf{v}. \tag{1}$$

At initial time t_0, it can be written that: $\rho\,(t_0, \mathbf{x}) = \rho_0\, \mathbf{1}_{\mathcal{D}_0}(\mathbf{x})$ where $\mathbf{1}_{\mathcal{D}_0}(\mathbf{x})$ is the indicator function of the subset $\mathcal{D}_0$.

The impact of crowd density for a standing crowd and a moving crowd is important to understand for crowd safety. In the UK Guides produced to advise on crowd safety issues (cf. [19]), the safety limit for crowd density is stated as 4 pedestrians per square meter for a moving crowd and 4.7 for standing areas. To be closer to reality, the individual dimensions of pedestrians must be taken into account in the density analysis. For a totally packed metro train (French RATP), the density is between 6 and 8 pedestrians/m^2. In conclusion, a maximum value ρ_{max} for local crowd density, $\rho_{max} \leq 8$ pedestrians/m^2, is introduced, and a maximum number of pedestrians N_{max} is then deduced: $N_{max} = \pi\, r^2\, \rho_{max}$.

In our model, dimensionless quantities are preferred. To do that, from the following reference variables:

- L: a characteristic length of the domain Ω, for example its diagonal when Ω is rectangle shaped,
- V_m: the maximum speed of the pedestrian walking unobstructed in the environment,