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Blow-up analysis in quasilinear reaction–diffusion problems with weighted nonlocal source

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ABSTRACT

In this paper, we consider the blow-up of solutions to a class of quasilinear reaction–diffusion problems

$$\begin{cases} (g(u))_t = \nabla\cdot\left(\rho\left(|\nabla u|^2\right)\nabla u\right) + a(x)f(u) & \text{in } \Omega\times(0,t^*),\\ \dfrac{\partial u}{\partial \nu} + \gamma u = 0 & \text{on } \partial\Omega\times(0,t^*),\\ u(x,0) = u_0(x) & \text{in } \overline{\Omega}, \end{cases}$$

where Ω is a bounded convex domain in $\mathbb{R}^n (n \geq 2)$, weighted nonlocal source satisfies $a(x)f(u(x,t)) \leq a_1 + a_2(u(x,t))^p \left(\int_\Omega (u(x,t))^l \mathrm{d}x\right)^m$, and $a_1, a_2, p, l,$ and m are positive constants. By utilizing a differential inequality technique and maximum principles, we establish conditions to guarantee that the solution remains global or blows up in a finite time. Moreover, an upper and a lower bound for blow-up time are derived. Furthermore, two examples are given to illustrate the applications of obtained results.

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1. Introduction

The mathematical studies of the blow-up phenomena of solutions to reaction–diffusion equations and systems have received extensive attention during the last few decades. For results of this area, we refer to [1–4]. Researchers usually considered the reaction–diffusion problems with local source, gave the general conditions which imply the solution blows up, and derived the estimates of the bounds for blow-up time of the solution (see [5–14]). However, a growing number of physical phenomena have been formulated into nonlocal mathematical models in recent years. It seems that nonlocal models are more near to practical problems than local models in a sense. To nonlocal models, many local theories seem no longer valid. The investigations of nonlocal problems become more difficult and challenging. In this paper, we deal with the following reaction–diffusion problems with weighted nonlocal source:

$$\begin{cases} (g(u))_t = \nabla\cdot(\rho(|\nabla u|^2)\nabla u) + a(x)f(u) & \text{in } \Omega\times(0,t^*),\\ \dfrac{\partial u}{\partial \nu} + \gamma u = 0 & \text{on } \partial\Omega\times(0,t^*),\\ u(x,0) = u_0(x) & \text{in } \overline{\Omega}. \end{cases} \tag{1.1}$$

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In problems (1.1), $\Omega \subset \mathbb{R}^n$ $(n \geq 2)$ is a bounded convex domain with sufficiently smooth boundary $\partial\Omega$, weighted nonlocal source satisfies

$$a(x)f(u(x,t)) \leq a_1 + a_2(u(x,t))^p \left(\int_\Omega (u(x,t))^l \mathrm{d}x \right)^m, \tag{1.2}$$

a_1, a_2, p, l, and m are positive constants, $\frac{\partial u}{\partial \nu}$ denotes the outward normal derivative on $\partial\Omega$, t^* denotes the maximal existence time of u, and γ is a positive constant. Let $\mathbb{R}_+ = (0, +\infty)$. Throughout this paper, we suppose that g is a $C^2(\overline{\mathbb{R}}_+)$ function and $g'(s) > 0$ with all $s > 0$, ρ is a positive $C^2(\overline{\mathbb{R}}_+)$ function satisfying $\rho(s) + 2s\rho'(s) > 0$ for $s > 0$, a is a positive $C(\overline{\Omega})$ function, f is a nonnegative $C^1(\mathbb{R}_+)$ function, and u_0 is a positive $C^2(\overline{\Omega})$ function and satisfies compatibility condition. From maximum principles [15], we know that classical solution u of (1.1) is positive in $\overline{\Omega} \times [0, t^*)$. Moreover, regularity theorem [16] guarantees $u \in C^3(\Omega \times (0, t^*)) \cap C^2(\overline{\Omega} \times [0, t^*))$.

For the information about the nonlocal problems of reaction–diffusion equations, we refer readers to [17–21] and the references therein. Especially, some nonlocal problems with weighted source were investigated in [18]. Fang and Ma [18] concerned with the following problems:

$$\begin{cases} u_t = \sum_{i,j=1}^n (a^{ij}(x)u_{x_i})_{xj} - a(x)f(u), & \text{in } \Omega \times (0, t^*), \\ \sum_{i,j=1}^n a^{ij}(x)u_{x_i}\nu_j = g(u) & \text{on } \partial\Omega \times (0, t^*), \\ u(x, 0) = u_0(x) & \text{in } \overline{\Omega}, \end{cases}$$

where $\Omega \subset \mathbb{R}^n (n \geq 2)$ is a bounded star-shaped domain with smooth boundary $\partial\Omega$, nonlocal source satisfies

$$f(u(x,t)) \geq a_2(u(x,t))^p \left(\int_\Omega (u(x,t))^l \mathrm{d}x \right)^m,$$

and a_2, p, l, and m are positive constants. They derived conditions which imply the solution blows up in finite time or exists globally. Moreover, upper and lower bounds for blow-up time are obtained. Their approach depends heavily upon using a differential inequality technique.

As far as we know, there is little information on the blow-up phenomena of problem (1.1). Motivated by the above works, we study the blow-up problems of (1.1). It seems that auxiliary functions defined in [18] are no longer applicable to (1.1). In order to achieve our purpose, we need to construct completely different auxiliary functions with those in [18]. Besides, we find that it is impossible to use only the differential inequality technique to study the problem (1.1). Hence, we combine maximum principles and the differential inequality technique to deal with (1.1).

The outline of this paper is as follows. In Section 2, when $\Omega \subset \mathbb{R}^n$ $(n \geq 2)$, we obtain a criterion for blow-up of the solution of problem (1.1) and lead to an upper bound for blow-up time. In Section 3, when $\Omega \subset \mathbb{R}^n$ $(n \geq 3)$, we derive a lower bound for blow-up time. Section 4 is devoted to getting conditions to ensure that the solution remains global when $\Omega \subset \mathbb{R}^n$ $(n \geq 2)$. In Section 5, two examples are given to illustrate the abstract results of this paper.

2. The blow-up solution

In this section, we obtain a criterion for blow-up to occur and derive an upper bound for blow-up time. We start with Lemma 2.1 which can be derived by using maximum principles.

Lemma 2.1. *Let u be a classical solution of (1.1). Assume*

$$\nabla \cdot \left(\rho \left(|\nabla u_0|^2 \right) \nabla u_0 \right) + a(x)f(u_0) \geq 0 \ \text{ in } \overline{\Omega}. \tag{2.1}$$

Then

$$u_t(x,t) \geq 0 \ \text{ in } \overline{\Omega} \times [0, t^*).$$

Proof. We construct auxiliary function

$$\xi(x,t) = u_t(x,t)$$

and compute

$$\begin{aligned} \xi_t(x,t) &= u_{tt}(x,t) \\ &= \left(\frac{1}{g'(u)} \left[\nabla \cdot \left(\rho \left(|\nabla u|^2 \right) \nabla u \right) + a(x)f(u) \right] \right)_t = \left(\frac{\rho}{g'} \Delta u + 2\frac{\rho'}{g'} \sum_{i,j=1}^n u_{x_i}u_{x_j}u_{x_ix_j} + \frac{af}{g'} \right)_t \end{aligned}$$

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