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# PROTEUS: A coupled iterative force-correction immersed-boundary cascaded lattice Boltzmann solver for moving and deformable boundary applications

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ABSTRACT

Many realistic fluid flow problems are characterised by high Reynolds numbers and complex moving or deformable geometries. In our previous study, we presented a novel coupling between an iterative force-correction immersed boundary and a multi-domain cascaded lattice Boltzmann method, Falagkaris et al., and investigated flows around rigid bodies at Reynolds numbers up to $10^5$. Here, we extend its application to flows around moving and deformable bodies with prescribed motions. Emphasis is given on the influence of the internal mass on the computation of the aerodynamic forces including deforming boundary applications where the rigid body approximation is no longer valid. Both the rigid body and the internal Lagrangian points approximations are examined. The resulting solver has been applied to viscous flows around an in-line oscillating cylinder, a pitching foil, a plunging SD7003 airfoil and a plunging and flapping NACA-0014 airfoil. Good agreement with experimental results and other numerical schemes has been obtained. It is shown that the internal Lagrangian points approximation accurately captures the internal mass effects in linear and angular motions, as well as in deforming motions, at Reynolds numbers up to $4 \cdot 10^4$. In all cases, the aerodynamic loads are significantly affected by the internal fluid forces.



## 1. Introduction

One of the challenging and important issues in computational fluid dynamics (CFD) is the accurate and efficient treatment of complex moving or deformable boundaries. This study will focus on the effect of the internal mass on the aerodynamic forces acting on a moving body, using a coupled immersed boundary cascaded lattice Boltzmann solver.

The lattice Boltzmann method (LBM) has been developed into a promising numerical scheme for simulating viscous fluid flows and has been successfully applied on both rigid and moving boundary applications. The consistency of the LBM with regard to the Navier–Stokes equations (NSE) has been established through various methods in the literature [1–8] and has been applied to many moving boundary simulations [9,10] because of its computational efficiency, simplicity and scalable parallel nature. In this work, the cascaded lattice Boltzmann method (CLBM), recently introduced by Geier et al., [11,12], is used for the fluid flow simulation due to its superior stability properties and higher degree of Galilean invariance over other lattice Boltzmann schemes [13–18].

In most practical fluid flow problems involving complex moving or deformable boundaries, non-uniform grid or body-fitted methods have been commonly used in the literature. Such treatments however, involve complicated algorithms with

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high computational costs and the solution accuracy is not as high as in uniform grid solvers. Decoupling the solution of the governing fluid equations from the implementation of the boundary conditions can simplify the solution process. The immersed boundary method (IBM), which was introduced by Peskin [19] in the 1970s to simulate blood flows in the human heart, has recently received great attention in simulating flows with complex moving geometries on a fixed Cartesian grid [20–23].

In IBM, the physical boundary is represented by a set of Lagrangian points, independent of the fixed Cartesian grid points. The no-slip condition on the boundary is enforced by applying body forces near the boundary resulting in movement of the internal to the body fluid. The motion of the internal fluid does not affect the flow characteristics outside the boundary. However, as pointed out by Suzuki and Inamuro [24], if the forces acting on the boundary are obtained by the negative sum of the body forces, as in [25], they are influenced by the motion of the internal mass. There have been only a few studies on the effect of the internal mass on the computation of the aerodynamic forces [26–28]. Uhlmann [27] treated the internal mass as a rigid body, imposing, however, a limitation on the density ratio of the body to the fluid for a stable simulation. Uhlmann [27] also computed the effect of the internal mass by summing the forces over all internal Eulerian points. However, the identification of the internal Eulerian points in a moving boundary application is very complex and computationally expensive. A similar observation on the limitation of the density ratio of the body to the fluid has been made by Ladd and Verberg [29]. Feng and Michaelides [28] further developed the work of Uhlmann [27] by eliminating the limitation on the density ratio. Shen et al. [30], following the work of Balaras [31], investigated the effect of the internal mass by integrating the NSE with the body forces. The magnitude of the internal mass effects was examined in systems with prescribed boundary motions. Most recently, Suzuki and Inamuro [24] proposed the Lagrangian point approximation (LPA) as an efficient method for describing the internal mass effects and compared it with the previous methods of Uhlmann [27] and Feng and Michaelides [28]. They examined the effect of the internal mass in cases where the body motion is defined by the fluid flow as well as, the dependency of the effect on very low Reynolds number flows. In the present study, the Lagrangian point approximation of Suzuki and Inamuro [24] is used. We focus on identifying the optimal configuration between arrangement of Lagrangian points and computational efficiency, as well as on examining the accuracy of the method at high Reynolds number flows around deformable boundaries for the first time.

In recent years, many efforts have been made in order to improve the coupling between the IBM and the LBM. In the penalty method, proposed by Feng and Michaelides [9], the immersed boundary is allowed to deform slightly and is restored back to its target position using a linear spring approximation. Dupuis et al. [32] presented a direct-forcing IBM, where the boundary force is computed using the interpolated velocity and a desired reference velocity. The momentum exchange of the particle distributions at the boundary was used by Niu et al. [33] to calculate the force acting on the immersed boundary. However, the non-slip boundary condition cannot be satisfied exactly by those methods. A few iterative IB schemes exist in the literature [34,35] that improve the accuracy of the prescribed boundary conditions. Zhang et al. [36] proposed an iterative force correction scheme based on Cheng's external forcing term [37]. Wu and Shu [38] developed an implicit velocity correction-based IB-LBM based on Guo's external forcing term [39]. In the present study, the iterative force correction IB scheme proposed by Zhang et al. [36] is used as described in our previous study [40].

This paper focuses on the effect of the internal mass in the computation of the aerodynamic forces for viscous fluid flows around moving and deformable boundaries at a wide range of Reynolds numbers. The coupled iterative direct forcing immersed boundary cascaded lattice Boltzmann method (IDF-CLBM), presented in our previous study [40], is used for the viscous fluid flow simulations. The effect of the internal mass is investigated using the Lagrangian point approximation method. First, the effect on the drag force is examined by considering the flow around a bluff body (circular cylinder) oscillating in a stationary fluid. Second, the effect on the lift force is examined by investigating the flow around streamlined bodies (foils) undergoing pitching and heaving motions. Finally, the effect on both the lift and drag forces of a plunging and deformable foil is investigated. The paper is organised as follows. In Section 2, the numerical method is presented. That includes the central moment formulation of the LBM, the iterative direct forcing IBM and the LPA treatment of the internal mass. Numerical results and the accuracy and robustness of the proposed scheme are reported in Section 3. Finally, the conclusions are summarised in Section 4.

## 2. Numerical method

### 2.1. The cascaded lattice Boltzmann method

#### 2.1.1. Central-moment lattice Boltzmann formulation

Consider a two-dimensional athermal fluid at a Cartesian coordinate system $(x, y)$ and let only the density $\rho(x, y)$, the velocity $\mathbf{u} = (u_x, u_y)$, and the external forces $\mathbf{F}(x, y)$ to characterise its local hydrodynamic behaviour. The nine-velocity square lattice model, denoted as D2Q9 [14], has been successfully used in the literature [41] for two-dimensional flows. Under the presence of external forces, the discrete evolution equation for the CLBM may be written as

$$f_\alpha(\mathbf{x} + \mathbf{e}_\alpha \delta t, t + \delta t) = f_\alpha(\mathbf{x}, t) + \Omega_a(\mathbf{x}, t) + \delta t S_a, \tag{1}$$

where $\mathbf{e}_\alpha : \alpha = 0, 1, \ldots, 8$ is the discrete velocity set; $f_\alpha(\mathbf{x}, t) : \alpha = 0, 1, \ldots, 8$ are the discrete particle distribution functions (PDF) at time $t$ and position $\mathbf{x}$; $\Omega_\alpha(\mathbf{x}, t) : \alpha = 0, 1, \ldots, 8$ is the discrete collision operator, and $S_\alpha(\mathbf{x}, t) : \alpha = 0, 1, \ldots, 8$ are the discrete forcing terms. Using the transformation matrix $\mathbf{K}$ [42,12] the collision operator takes the form

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