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Laminar fluid flow and heat transfer in an internally corrugated tube by means of the method of fundamental solutions and radial basis functions

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ABSTRACT

The paper shows application of the method of fundamental solutions in combination with the radial basis functions for analysis of fluid flow and heat transfer in an internally corrugated tube. Cross-section of such a tube is mathematically described by a cosine function and it can potentially represent a natural duct with internal corrugations, e.g. inside arteries. The boundary value problem is described by two partial differential equations (one for fluid flow problem and one for heat transfer problem) and appropriate boundary conditions. During solving this boundary value problem the average fluid velocity and average fluid temperature are calculated numerically. In the paper the Nusselt number and the product of friction factor and Reynolds number are presented for some selected geometrical parameters (the number and amplitude of corrugations). It is shown that for a given number of corrugations a minimal value of the product of friction factor and Reynolds number can be found. As it was expected the Nusselt number increases with increasing amplitude and number of corrugations.

1. Introduction

In the nature many structures are corrugated (sometimes called in the literature wavy structures). Also many fluids flow through corrugated ducts in the nature, e.g. flow induced by rhythmic wall contractions in insect's tracheal tubes [1], peristaltic flow [2,3] or flow through arterial stenoses [4]. In the literature many researchers analyzed numerically longitudinal or transverse flows between corrugated walls. Wang analyzed longitudinal flow between the corrugated plates using the perturbation technique [5,6]. The same author employed the perturbation method to analyze three-dimensional flow induced by relative motion of two corrugated plates [7]. The same method was employed by Ng and Wang to consider Darcy–Brinkman flow in such a duct [8]. Non-Newtonian fluids (power-law and Carreau fluids) longitudinal flows was considered by means of meshless methods by Grabski and Kołodziej [9,10]. Many authors examined also transverse flow between wavy walls, e.g. in [11,12]. In modeling of arterial blood flow many researchers considered transverse flow through wavy walls. The authors studied effect of irregularities in such ducts on blood flow [13,14]. In the paper influence of the corrugations in cross-sections is considered (see Fig. 1). To the best knowledge of the authors such a duct was not considered as yet in the literature on modeling of blood flow.

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From the other hand the internally corrugated tube can be treated as a special case of internally finned duct with smooth fins, described by a cosine function. In the literature one can find many different geometries of internally finned tubes, circular tube with e.g. straight fins [15], triangular fins [16] or parabolic fins [17], etc. In the literature to analyze fluid flow and heat transfer in such tubes common numerical methods has been applied, e.g. the finite element method [18], the finite volume method [19] or the finite difference method [15]. Application of meshless methods in these problems is rather new. To the best knowledge of the authors only two papers have been published as yet on application of the method of fundamental solutions with radial basis functions for analysis of fluid flow and heat transfer in an internally finned tube [20,21].

More and more researchers pay their attention to meshless methods [22–24]. The methods have been applied for analysis many scientific and engineering problems in the literature. One of such methods is the method of fundamental solutions (MFS). In the method the approximate solution is a linear combination of fundamental solutions which are functions of distance between any point inside the considered domain and the source point located outside the domain, on so called pseudo-boundary (or fictitious boundary). The unknown coefficients are determined using the boundary collocation technique [25]. The MFS is quite easy to implement and the main question in the method is how to distribute the source points (what should be the shape of the pseudo-boundary and what should be the distance between the pseudo-boundary and the considered domain), see e.g. [26–28]. The MFS was proposed in 1964 by Kupradze and Aleksidze [29]. Probably the first numerical implementation of the method was shown by Mathon and Johnston in 1977 [30]. Review of applications of the MFS for analysis various problems can be found e.g. in [31,32].

The MFS has been successfully employed by many authors to analyze different problems in fluid mechanics. Most of these applications are related to Stokes flow in different formulations, e.g. stream function formulation [33–35] or pressure–velocity formulation [36–38]. Some authors considered also potential flows using the MFS, e.g. axisymmetric potential flows [39] or potential flows for ship motion prediction [40]. Young et al. proposed a numerical procedure based on the MFS for solving the Navier–Stokes equations [41]. The MFS has been applied also to some others fluid mechanics problems, e.g. flows in porous media [42], slow Brinkman flow [43], unsteady Burgers' equations [44], Oseen flows [45,46], sloshing phenomenon [47] or steady-state groundwater flow [48]. In the literature one can find studies on longitudinal flows through different shapes of ducts by means of the MFS, e.g. flow in internally finned duct [20] or between corrugated plates [9,10]. A study of longitudinal flow is done also in the paper, where flow through an internally corrugated tube is considered. In addition, heat transfer in such a duct is also analyzed.

Application of the MFS for nonlinear problems in the literature in most cases is based on the Picard iteration method or the homotopy analysis method [49]. These two methods were compared for different boundary value problems by Uściłowska [50]. In both cases the nonlinear problem is transformed into a sequence of inhomogeneous problems. In the paper the Picard iteration method is employed. Then the governing equation is divided into the linear term and the nonlinear term. In order to transform the nonlinear problem into a sequence of inhomogeneous problems, at each iteration the approximate solution from the previous iteration step is applied in the nonlinear term. In such a way at each iteration step the inhomogeneous problem is to solve.

The MFS for inhomogeneous problems has been successfully applied in many papers. In the literature there are two main approaches for solving inhomogeneous problems using the MFS. In both methods the approximate solution consists of the general solution and the particular solution. The general solution is obtained using the MFS. The methods differ in obtaining the particular solution. These two methods – one based on the Newtonian potential [51] and the second one is based on the dual reciprocity method (DRM) [52] – were applied in combination with the MFS and the results were compared by Golberg [53]. He showed that the second approach gives more accurate results. In different applications of the DRM in combination with the MFS for solving inhomogeneous problems the radial basis functions (RBFs) are used very often to interpolate the inhomogeneous term. This procedure has been employed previously in many problems in the literature, e.g. isothermal gas flow in porous medium [54], thermoelasticity of functionally graded materials [55] or heat conduction in anisotropic and inhomogeneous media [56].

In the paper an alternative way to obtain the particular solution is proposed. It can be done using the global radial basis function collocation method (GRBFCM). It is called also the Kansa method in the literature because Edward Kansa proposed the method in 1990 for solving some boundary value problems [57]. In the method the approximate solution is a linear combination of RBFs. The method has been successfully applied in many scientific and engineering problems, e.g. in two-dimensional or three-dimensional Stokes flows [58] or in planetary scale flows [59]. However application of the GRBFCM for obtaining the particular solution is not so common in the literature. In the GRBFCM there is no need to know the particular solution of the RBF (which is typical for application of the DRM with RBFs for obtaining the particular solution of inhomogeneous problem) but just to know appropriate derivatives of the applied RBF. This approach seems to the authors much easier.

In the paper steady, fully-developed fluid flow and heat transfer in an internally corrugated tube is considered. Such a tube is quite new in the literature. This study is a continuation of the work presented during the 10th International Conference on Heat Transfer, Fluid Mechanics and Thermodynamics in 2014 [60]. However the numerical procedure is modified in this paper. The problem is described by one linear and one nonlinear governing equations with linear boundary conditions. At the beginning fluid flow problem is solved using the MFS. After that the average fluid velocity and product of friction factor and Reynolds number can be determined. The nonlinear problem describing heat transfer is solved using the Picard iteration method. In order to obtain the first approximation the right-hand side in the first step of the iteration process is modified.

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