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A new approach toward stabilization in a two-species chemotaxis model with logistic source

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ABSTRACT

This paper aims at providing an alternative approach to study global dynamic properties for a two-species chemotaxis model, with the main novelty being that both populations mutually compete with the other on account of the Lotka–Volterra dynamics. More precisely, we consider the following Neumann initial–boundary value problem

$$\begin{cases} u_t = d_1\Delta u - \chi_1\nabla\cdot(u\nabla w) + \mu_1 u(1-u-a_1 v), & x\in\Omega, \quad t>0,\\ v_t = d_2\Delta v - \chi_2\nabla\cdot(v\nabla w) + \mu_2 v(1-a_2 u - v), & x\in\Omega, \quad t>0,\\ 0 = d_3\Delta w - w + u + v, & x\in\Omega, \quad t>0, \end{cases}$$

in a bounded domain $\Omega\subset\mathbb{R}^n$, $n\geq 1$, with smooth boundary, where $d_1, d_2, d_3, \chi_1, \chi_2, \mu_1, \mu_2, a_1, a_2$ are positive constants.

When $a_1\in(0,1)$ and $a_2\in(0,1)$, it is shown that under some explicit largeness assumptions on the logistic growth coefficients μ_1 and μ_2, the corresponding Neumann initial–boundary value problem possesses a unique global bounded solution which moreover approaches a unique positive homogeneous steady state (u^*, v^*, w^*) of above system in the large time limit. The respective decay rate of this convergence is shown to be exponential.

When $a_1 \geq 1$ and $a_2\in(0,1)$, if μ_2 is suitable large, for all sufficiently regular nonnegative initial data u_0 and v_0 with $u_0\not\equiv 0$ and $v_0\not\equiv 0$, the globally bounded solution of above system will stabilize toward $(0,1,1)$ as $t\to\infty$ in algebraic.

1. Introduction

This paper deals with a classical two-species parabolic–parabolic– elliptic chemotaxis model

$$\begin{cases} u_t = d_1\Delta u - \chi_1\nabla\cdot(u\nabla w) + \mu_1 u(1-u-a_1 v), & x\in\Omega, \quad t>0,\\ v_t = d_2\Delta v - \chi_2\nabla\cdot(v\nabla w) + \mu_2 v(1-a_2 u - v), & x\in\Omega, \quad t>0,\\ 0 = d_3\Delta w - w + u + v, & x\in\Omega, \quad t>0,\\ \dfrac{\partial u}{\partial \nu} = \dfrac{\partial v}{\partial \nu} = \dfrac{\partial w}{\partial \nu} = 0, & x\in\partial\Omega, \quad t>0,\\ u(x,0)=u_0(x),\ v(x,0)=v_0(x), & x\in\Omega, \end{cases} \tag{1.1}$$

with positive parameters $d_1, d_2, d_3, \chi_1, \chi_2\ \mu_1, \mu_2, a_1$ and a_2, which models the spatio-temporal evolution of two populations that mutually compete according to the Lotka–Volterra dynamics, and in which all individuals move according to random

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diffusion and chemotaxis toward the same signal jointly produced by themselves. In (1.1), $u = u(x, t)$ and $v = v(x, t)$ denote the density of two different populations, respectively, and $w = w(x, t)$ represents the concentration of the chemoattractant. Furthermore, the motion of the chemoattractant w has been described by an elliptic equation since signals diffuse much faster than species.

The model (1.1) originates in the classical single-species chemotaxis system obtained by letting $v \equiv 0$ which is considered as a prototypical model for one population processes [1]. Without logistic term in the sense that $\mu_1 = 0$, it is known that the solution of the corresponding single-species chemotaxis is global in time and remains bounded if either $n = 1$ [2], or $n = 2$ and the initial data is suitable small [3,4], whereas when either $n = 2$ and the initial data is large [5–8], or $n \geq 3$ the solution will blow up [9,10]. When $\mu_1 > 0$, numerous simulations or theory researches on the general chemotaxis-growth system have revealed that blow-up phenomena can entirely be ruled out when $n \geq 2$ and $\mu_1 > 0$ is sufficiently large [11–15]. For instance, when $\mu_1 > ((n-2)_+/n)\chi_1$ then all solutions are global and bounded ([16], see [17–21] for the parabolic–parabolic counterpart). On the other hand, for suitable small $\mu_1 > 0$, the dampening role of logistic-type sources is yet far from understood in such single-species chemotaxis system, but that in such systems, quite colorful dynamical properties have been detected both numerically and also analytically. For example, the exceedance of corresponding carrying capacities seems possible (see [22–25]), and in presence of merely certain subquadratic (generalized) logistic-type dampenings, even finite-time blow up will occur (see [26–28]).

Generalizing above situation to two species that react onto the same chemical substances and assuming that the species mutually compete according to classical Lotka–Volterra kinetics, we can formulate the generalized two-species chemotaxis model (1.1) (see e.g. [29–31]). Accordingly, the results for this two-species chemotaxis system also inherit some important properties from the original single-species chemotaxis model. For instance, if $\mu_1 = 0$ and $\mu_2 = 0$ in (1.1), the smallness assumption on the total mass of species guarantees the global existence of the solution [32,33], whereas in both parabolic–parabolic–elliptic and fully parabolic versions of the latter blow up still occur when either $n = 2$ and the total mass is suitably large or $n \geq 3$ [32–38].

In cases when $\mu_1 > 0$ and $\mu_2 > 0$, model (1.1) also enjoys some global well-posedness and boundedness properties in appropriate frameworks. For the fully parabolic version of (1.1) obtained by replacing the third equation by $w_t = d_3\Delta w - w + u + v$, under some largeness conditions on the size of μ_1 and μ_2 as related to the chemotactic sensitivities χ_1 and χ_2 the authors in [39] established the global existence of classical solutions when $n \geq 1$ and $d_1 = d_2$. Later on by means of the construction of suitable energy functionals Bai et al. [40] showed the large time behavior of bounded solution for the two cases $a_1, a_2 \in (0, 1)$ and $a_1 \geq 1, a_2 \in (0, 1)$. Recently, Mizukami [41] extended these results to more particular situations (see also [42]). When there is no competition in the sense that $a_1 = a_2 = 0$, Negreanu et al. [43,44] established the global existence and global behavior for small diffusion rates, where the smallness was later removed in [45]. We also refer to [46–48] for the research on steady states and refer to [49–51] for nonlinear chemotaxis sensitivity functions. For the parabolic–parabolic–elliptic system (1.1), under explicit conditions on large number of parameters the global dynamical properties of the two cases $a_1, a_2 \in (0, 1)$ and $a_1 > 1, a_2 \in (0, 1)$ are also discussed in [52,53] and [54], separately. The proof of above convergence results is based on certain comparison arguments, which is a powerful tool and applicable for many parabolic–elliptic models (see [55,56], for instance). However, in this situation it seems to be hard to give convergence rates, unlike the parabolic–parabolic–parabolic case in [40].

Main Results. The purpose of this paper is to provide an alternative approach (by constructing energy functionals) to investigate the asymptotic behavior of solutions of (1.1) under simple choices of parameters, which further studied the influence of chemotaxis effects on the stability of the homogeneous steady states determined by the competitive terms. Moreover, we will further give the respective convergence rates.

A necessary first step then consists in establishing the global boundedness of solutions to (1.1) by means of the standard a priori estimates under some largeness conditions on the size of μ_1 and μ_2.

Theorem 1.1. *Suppose that $\Omega \subset \mathbb{R}^n (n \geq 1)$ be an arbitrary smooth bounded domain, and that $u_0 \in C^0(\bar{\Omega})$ and $v_0 \in C^0(\bar{\Omega})$ with $u_0 \not\equiv 0$ and $v_0 \not\equiv 0$ are nonnegative. If $d_1, d_2, d_3, \chi_1, \chi_2, \mu_1, \mu_2, a_1$ and a_2 are positive and satisfy*

$$q_1 + q_2 < d_3, \tag{1.2}$$

where $q_1 = \frac{\chi_1}{\mu_1}$ and $q_2 = \frac{\chi_2}{\mu_2}$. Then the system (1.1) possesses a unique global classical solution which is bounded in the sense that there exists $C > 0$ such that

$$\|u(\cdot, t)\|_{L^\infty(\Omega)} + \|v(\cdot, t)\|_{L^\infty(\Omega)} + \|w(\cdot, t)\|_{W^{1,\infty}(\Omega)} \leq C \;\; \textit{for all} \;\; t > 0. \tag{1.3}$$

Remark 1.1. When $n \leq 2$, by constructing some inequalities involving the logarithmic entropy $\int_\Omega u \ln u$ (see [39,40]), arbitrarily small μ_1 and μ_2 could enforce the global boundedness of solutions of (1.1).

Remark 1.2. If $d_3 = 1$, the condition (1.2) can be reduced to

$$q_1 + q_2 < 1, \tag{1.4}$$

which is consistent with the results in [54].