ARTICLE IN PRESS

Computers and Mathematics with Applications ▮ (▮▮▮▮) ▮▮▮–▮▮▮

Contents lists available at ScienceDirect

Computers and Mathematics with Applications

journal homepage: www.elsevier.com/locate/camwa

Auxiliary equation method for time-fractional differential equations with conformable derivative

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ARTICLE INFO

Article history:
Received 21 April 2017
Received in revised form 17 July 2017
Accepted 15 October 2017
Available online xxxx

Keywords:
Conformable fractional derivative
Auxiliary equation method
Time-fractional differential equations

ABSTRACT

In this paper, the auxiliary equation method is applied to obtain analytical solutions of $(2+1)$-dimensional time-fractional Zoomeron equation and the time-fractional third order modified KdV equation in the sense of the conformable fractional derivative. Given equations are converted to the nonlinear ordinary differential equations of integer order; and then, the resulting equations are solved using a novel analytical method called the auxiliary equation method. As a result, some exact solutions for them are successfully established. The exact solutions obtained by the proposed method indicate that the approach is easy to implement and effective.

1. Introduction

The fractional calculus, which was introduced by Leibnitz in 1695, has attracted much attention. The impact of this fractional calculus in both pure and applied science and engineering disciplines started to increase substantially during the past twenty years seemingly [1,2].

Noticing remarkable various applications of fractional derivative, it is then apparent to search proper definition and method of solutions to the resulted fractional differential equations. Accordingly, many proposed definitions from different point of view of applications have been introduced in the literature such as Hilfer, Riemann–Liouville, Caputo and so on.

The exact solutions of nonlinear fractional differential equations play a crucial role in the mathematical physics. It is used to describe the observed various qualitative and quantitative features of nonlinear phenomenons in many fields of mathematical physics and nonlinear sciences. The availability of symbolic computation packages can facilitate many direct and powerful approaches to establish exact solutions to fractional differential equations. Researchers have presented various methods to search exact solutions such as modified simple equation method [3,4], trial equation method [5–7], first integral method [8–12], (G'/G)—expansion method [13,14], sub equation method [15], exp-function method [16], homotopy perturbation method [17], homotopy analysis method [18], generalized Kudrayshov method [19–21], functional variable method [22] and so on [23–25].

In [26], a new approach has been proposed based on the extended tanh-function method which was called a new auxiliary equation method. By using this method, Sirendaoreji obtained some new exact travelling wave solutions with the constraint $c = 0$ of some nonlinear partial differential equations [26–29]. This approach has been applied to nonlinear fractional differential equations with Riemann–Liouville derivative [30]. In this paper, our goal is to apply the algorithm to nonlinear fractional differential equations with conformable fractional derivative.

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https://doi.org/10.1016/j.camwa.2017.10.016
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2. Brief of conformable fractional derivative

Recently, the authors Khalil et al. introduced a new simple and intriguing definition of the fractional derivative called conformable fractional derivative [31]. This derivative is well-behaved and obeys the Leibniz rule and chain rule. Here we give the definition of the conformable fractional derivative and some useful properties of this new derivative [32].

Definition 1. Suppose $f : [0, \infty) \to R$ be a function. Then, the conformable fractional derivative of f of order α, $0 < \alpha \le 1$, is defined as

$$(T_\alpha f)(t) = \lim_{\varepsilon \to 0} \frac{f(t + \varepsilon t^{1-\alpha}) - f(t)}{\varepsilon}$$

for all $t > 0$. Some useful properties can be listed as follows:

1. Linearity: $T_\alpha(af + bg) = a(T_\alpha f) + b(T_\alpha g)$, for all $a, b \in R$
2. Leibniz rule: $T_\alpha(fg) = fT_\alpha(g) + gT_\alpha(f)$
3. Chain rule: Let $f : (0, \infty) \to R$ be a differentiable and $\alpha-$differentiable function, g be a differentiable function defined in the range of f.

$$T_\alpha(f \circ g)(t) = t^{1-\alpha} g'(t) f'(g(t)).$$

Moreover, the following rules hold.
$T_\alpha(t^p) = pt^{p-\alpha}$, for all $p \in R$
$T_\alpha(\lambda) = 0$, for all constant functions $f(t) = \lambda$
$T_\alpha(f/g) = \frac{g(T_\alpha f) - f(T_\alpha g)}{g^2}$.
Additively, if f is differentiable, then $T_\alpha(f)(t) = t^{1-\alpha} \frac{df}{dt}(t)$.

3. Auxiliary equation method

Suppose that we have a nonlinear fractional differential equation with the conformable time-fractional derivative, say in the independent variables x and t and dependent variable u, given by

$$F\left(u, \frac{\partial^\alpha u}{\partial t^\alpha}, \frac{\partial u}{\partial x}, \frac{\partial^{2\alpha} u}{\partial t^{2\alpha}}, \frac{\partial^2 u}{\partial x^2}, \ldots\right) = 0, \tag{1}$$

where F is a polynomial of $u(x, t)$ and its various partial conformable derivatives including the highest order derivatives and nonlinear terms.

First, by using the new definition for travelling wave variable

$$u(x, t) = u(\xi),\ \xi = kx - l\frac{t^\alpha}{\alpha}, \tag{2}$$

where k and l are nonzero arbitrary constants to be determined later, Eq. (1) can be rewritten as a nonlinear ordinary differential equation (ODE) as follows:

$$Q(u, u', u'', \ldots) = 0. \tag{3}$$

According to the algorithm, we can seek for the solutions of Eq. (3) in the form:

$$u(\xi) = \sum_{i=0}^{2N} a_i F^i(\xi), \tag{4}$$

where $a_i (i = 0, \ldots, 2N)$ are constants to be determined later and the positive integer N can be determined by using homogeneous balance method between the highest order derivatives and the nonlinear terms appearing in Eq. (3). Here $F(\xi)$ satisfies the following variable separated ordinary differential equation:

$$(F')^2(\xi) = aF^2(\xi) + bF^4(\xi) + cF^6(\xi), \tag{5}$$

where a, b, c are parameters to be determined.

The positive integer N can be determined by considering the homogeneous balance between the highest order derivative term with the highest order nonlinear term appearing in Eq. (3). Then by substituting Eq. (4) along with Eq. (5) into Eq. (3) and equating the coefficients of all powers of $F(\xi)$ to zero yields a set of algebraic equations for unknowns $a, b, c; a_i (i = 0, \ldots, 2N)$, k and l. We find the solutions of the set of algebraic equations by Maple and substitute the obtained solutions in this step back into Eq. (3) so as to obtain the exact solutions of Eq. (1) [33–35].

4. Applications

In this section, we have dealt with two fractional differential equations as an application of the auxiliary equation method.