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Solitary waves, homoclinic breather waves and rogue waves of the $(3+1)$-dimensional Hirota bilinear equation[☆]

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ABSTRACT

In this paper, the $(3+1)$-dimensional Hirota bilinear equation is investigated, which can be used to describe the nonlinear dynamic behavior in physics. By using the Bell polynomials, the bilinear form of the equation is derived in a very natural way. Based on the resulting bilinear form, its N-solitary waves are further obtained by using the Hirota's bilinear theory. Finally, by using the Homoclinic test method, we obtain its rational breather wave and rogue wave solutions, respectively. In order to better understand the dynamical behaviors of the equation, some graphical analyses are discussed for these exact solutions.

1. Introduction

It is great significance to investigate the exact solutions of nonlinear evolution equations (NLEEs), which always play an important role in nonlinear science fields, especially in nonlinear mathematical physical science. Recently, there are many methods to find exact solutions of NLEEs, such as Darboux and Bäcklund transformation (BT) [1], inverse scattering transformation [2], Hirota bilinear method [3], Lie group method [4,5], etc. In recent years, rogue waves, as a special type of nonlinear waves, also known as monster waves, killer waves, extreme waves, giant waves, have aroused people's interest in different branches of physics, such as plasmas, deep ocean, nonlinear optic, Bose–Einstein condensates, biophysics, superfluids, financial markets and other related fields [6–26]. In contrast to tsunamis and storms associated with typhoons that can be predicted hours (sometimes days) in advance, the rogue waves of ocean waves suddenly appear from nowhere and quickly disappear without a trace.

The aim of this paper is to study the solitary waves, homoclinic breather wave and rogue wave solutions of the following $(3+1)$-dimensional Hirota bilinear (HB) equation [27–29]

$$u_{yt} - u_{xxxy} - 3(u_x u_y)_x - 3u_{xx} + 3u_{zz} = 0, \tag{1.1}$$

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where $u = u(t,x,y,z)$ is a differentiable function. The HB equation (1.1) is completely integrable and possesses the N-soliton solutions. Under the transformation $t \to -T$, $x \to X$, $y \to X$, $z \to X$, and $u_x \to U$, Eq. (1.1) reduces to the classical Korteweg–de Vries (KdV) equation

$$U_T + U_{XXX} + 6UU_X = 0, \tag{1.2}$$

which is a mathematical model of waves on shallow water surfaces. As an extension of the KdV equation, the HB equation (1.1) admits the similar physical meaning as one of the KdV equation. The HB equation (1.1) can be used to describe the dynamics of solitons and nonlinear waves in fluid dynamics, plasma physics, and weakly dispersive media, etc. Moreover, several classes of exact solutions for Eq. (1.1) were discussed in [27–29]. In [27], applying the linear superposition principle to the Hirota bilinear equation (1.1), authors found two types of resonant multiple wave solutions. In [28], Lü and Ma derived its lump solutions to two types of dimensional reductions with $z = y$ and $z = t$. They also provided the sufficient and necessary conditions to guarantee analyticity and rational localization of the solutions. Moreover, they analyzed localized characteristics and energy distribution of these lump solutions. The lump solutions of dimensional reductions with $z = x$ for the HB equation (1.1) were also studied in [29]. However, there are very few work to study other dynamic behavior of Eq. (1.1).

To the best of our knowledge, much work have been done for Eq. (1.1), but rogue waves and breather waves of Eq. (1.1) have not been investigated before. The main purpose of the present paper is to employ an effective way to construct the rogue wave, the kink and rational breather wave solutions of Eq. (1.1). Besides, its bilinear form and N-soliton solutions are also derived by using the Bell polynomial.

The structure of this paper is given as follows. In Section 2, the bilinear representation of Eq. (1.1) is presented with a detailed derivation. In Section 3, we obtain its solitary wave with graphic analysis. In Section 4, we construct the rogue wave and breather wave of Eq. (1.1) by using the homoclinic breather limit method (HBLM) [30]. Finally, some conclusions and discussions are presented in Section 5.

2. Bilinear form

Let us consider the following transformation

$$u = d(t)q_x, \tag{2.1}$$

where $d = d(t)$ is a function to be determined later. Substituting Eq. (2.1) into Eq. (1.1), then integrating the resulting equation with respect to x, and taking the integral constant as zero, we have

$$E(q) = d(t)q_{yt} - (d(t)q_{xxxy} + 3d(t)^2 q_{xx}q_{xy}) - 3d(t)q_{xx} + 3d(t)q_{zz} = 0, \tag{2.2}$$

where E is a polynomial of q. Taking $d(t) = 1$, and using the results obtained in [31–38], we can get the form of the P-polynomials for Eq. (2.2) as follows

$$E(q) = P_{yt} - P_{3xy} - 3P_{2x} + 3q_{2z} = 0. \tag{2.3}$$

The above expression leads to the following bilinear equation

$$\left(D_tD_y - D_x^3D_y - 3D_x^2 + 3D_z^2\right) F \cdot F = 0, \tag{2.4}$$

with the aid of following transformation

$$q = 2\ln(F) \Longleftrightarrow u = dq_x = 2[\ln(F)]_x. \tag{2.5}$$

3. Solitary wave solutions

In order to construct the solitary wave solutions of Eq. (1.1). We expand F with respect to a formal expansion parameter as

$$F(x,y,z,t) = 1 + F^{(1)}\epsilon + F^{(2)}\epsilon^2 + F^{(3)}\epsilon^3 + F^{(4)}\epsilon^4 + \cdots . \tag{3.1}$$

Substituting Eq. (3.1) into Eq. (2.4), then equating the coefficients of all powers of ϵ equal to zero yields the following recursion relations for $F^{(n)}$

$$\begin{aligned}
&F_{yt}^{(1)} - F_{3xy}^{(1)} - 3F_{xx}^{(1)} + 3F_{zz}^{(1)} = 0,\\
&2\left(F_{yt}^{(2)} - F_{3xy}^{(2)} - 3F_{xx}^{(2)} + 3F_{zz}^{(2)}\right) = -\left(D_tD_y - D_x^3D_y - 3u_0D_x^2 - 3D_x^2 + 3D_z^2\right) F^{(1)} \cdot F^{(1)} = 0,\\
&F_{yt}^{(2)} - F_{3xy}^{(1)} - 3F_{xx}^{(2)} + 3F_{zz}^{(2)} = -\left(D_tD_y - D_x^3D_y - 3u_0D_x^2 - 3D_x^2 + 3D_z^2\right) F^{(1)} \cdot F^{(2)} = 0,
\end{aligned} \tag{3.2}$$

etc.