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Stock loan valuation based on the Finite Moment Log-Stable process

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ABSTRACT

The empirical test suggests that the log-return series of stock price in US market reject the normal distribution and admit instead a subclass of the asymmetric distribution. In this paper, we investigate the stock loan problem under the assumption that the return of stock follows the finite moment log-stable process (FMLS). In this case, the pricing model of stock loan can be described by a space-fractional partial differential equation with time-varying free boundary condition. Firstly, a penalty term is introduced to change the original problem to be defined on a fixed domain, and then a fully-implicit difference scheme has been developed. Secondly, based on the fully-implicit scheme, we prove that the stock loan value generated by the penalty method cannot fall below the value obtained when the stock loan is exercised early. Thirdly, the numerical experiments are carried out to demonstrate differences of stock loan model under the FMLS and the standard normal distribution. Optimal redemption strategy of stock loan has been achieved. Furthermore the impact of key parameters in our model on the stock loan evaluation are analyzed, and some reasonable explanation are given.

1. Introduction

In the contract of a stock loan, the investor can obtain a loan from the bank or other financial institution with their stocks as collateral and the contract gives investor a right, which called the value of a stock loan, that they can redeem the stocks at any valid time by repaying the principal and the loan interest. Since the stock loan can be used by risk aversion investors to transfer the risk of holding stocks to the financial institutions, and it also can establish market liquidity, so the theoretical models for pricing stock loan have been carried out.

Xia and Zhou [1] considered the pricing model for stock loan under the assumption that the dynamic evolution process of underlying follows the geometric Brownian motion. Since then, more and more researchers paid attentions to the problem of pricing stock loan with other conditions. Prominent examples including [2], in which it was assumed that there was a capped limit for the stock price when it exceeded a predetermined barrier in the contract. While, an automatic termination clause, cap and margin were added in the research paper [3]. Grasselli et al. [4] studied the stock loan pricing model in the incomplete markets. Under assumption that the risk-free interest rate follows the Rendleman–Bartter model without drift term, Chen et al. [5] considered the finite maturity stock loan pricing model. Cai et al. [6] investigated the pricing of both infinite and finite maturity stock loan under the hyper-exponential jump diffusion model.

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A common assumption kept in the existing literatures [2–6] is that the dynamic evolution process of risk asset follows the geometric Brownian motion. That is, the changes of asset price are independent and the log returns of underlying price follow the standard normal distribution. However, experimental investigations on asset return showed that discontinuities or jumps are believed to be an indispensable element of financial stock pricing (see e.g. [7–9]) and the distribution of asset return can appear "leptokurtic distribution" (see e.g. [10–14]). It turns out that many scholars used other stochastic process to characterize the abnormal fluctuations of stock price. For example, Merton [15] applied a jump diffusion process with Poisson jump to simulate the stock price. Sun [16] utilized a mixed fractional Brownian motion to capture the Non-normal feature of logarithmic returns of risk asset. Markose and Alentorn [17] suggested that the application of generalized extreme value (GEV) distribution to model the implied risk-neural density function provided a flexible framework that captures the negative skewness and excess kurtosis of return. A frequently used stochastic process called Finite Moment Log Stable (FMLS) process shows its strength in reflecting the jumps and "leptokurtic distribution" of risk asset returns. As a lévy process with maximum negative skewness, the FMLS exhibits a self-similarity or stability property, which means that the distribution of the α-stable motion over any horizon has the same shape upon scaling. With the tail index satisfying $\alpha < 2$, the risk-neutral distribution has fat tails over all horizons. Furthermore, the FMLS model can guarantee that all moments of risk-asset index levels are finite, which are needed for the existence of an equivalent martingale measure and finite financial derivatives value.

Through the Fourier transform technique, Cartea [18] derived the partial differential equation(PDE) governing European option price under the FMLS framework. It is worth noting that there is a fractional differential operator reflecting the non-localness caused by the pure jumps. Based on this work, many authors investigated financial derivatives pricing under FMLS framework. For instance, Chen et al. [19] obtained the analytical pricing formula of European option of the FMLS model by using Fourier integral transformation and Fox functions. Subsequently, they utilized the predictor–corrector method to analyze the American put option under FMLS model [20]. Marom and Momoniat [21] compared the Black–Merton–Scholes model under three kinds of Lévy process (FMLS, CGMY and KoBol). Zhang et al. [22] constructed an implicit finite difference scheme with second order accuracy for the European double barrier option under FMLS, CGMY and KoBol models, a fast bi-conjugate gradient stabilized method was employed to reduce the storage requirement and computational efforts. Chen [23] proposed a second order finite difference and power penalty methods to solve a space-fractional linear complementarity problem arising in the valuation of American option.

Due to the features of FMLS and it provides a better reference value to the holder of stock loan contract, we research the problem of stock loan with finite maturity under the framework of FMLS. In this case, the governing equation of the value of stock loan is a fractional partial differential equation (FPDE). Further the feature of early exercise at any valid time, results in an unknown free boundary. The non-local fractional operator and the nonlinearity associated with the free boundary add the difficulty of solving the fractional stock loan model.

In Ref. [20], a numerical approach with predictor–corrector method in the time direction and spectral collocation of high order in spatial discretization was provided to effectively price the American put options. Prior to doing so, a landau transform was introduced to shift the moving boundary conditions to fixed boundary conditions. While, we find the corresponding landau transform in Ref. [5] cannot work for the left-hand Riemann–Liouville fractional derivative when the stock loan model is defined on $(-\infty, x_f]$ with x_f being the moving boundary. Therefore, we consider a penalty method approach in which the free boundary is removed by adding a small and continuous penalty term to the FPDE. Consequently, the problem can be solved on a fixed domain. We set a fully-implicit scheme to obtain the numerical solution. Since the analytical solution is seldom available, so a fully-implicit scheme is employed. And based on this numerical scheme, we prove that the stock loan value generated by the penalty method cannot fall below the value obtained when the stock loan is exercised early, *i.e.* $V(x,t) \geq \max(e^x - Ke^{\gamma t}, 0)$. What is more, we plot the surface of $V(x,t) - \max(e^x - Ke^{\gamma t}, 0)$ with different time and assert price, which show that the stock loan value calculated by the penalty method is not less than the value obtained when the stock loan is exercised early.

The remainder of this paper is arranged as follows. In Section 2, we present the space-fractional differential equation model governing stock loan value, and the corresponding boundary and terminal conditions are appropriately proposed. In Section 3, by adding a penalty term, a fully nonlinear implicit difference scheme is developed to cope with the presence of the free-boundary and the scheme can be realized by Newton's iteration method. Then, an important property of the difference scheme has been proved, that is the numerical solution preserves the discrete form of inequality $V(x,t) \geq \max(e^x - Ke^{\gamma t}, 0)$. In Section 4, some numerical examples are carried out to demonstrate the strength and validity of our proposed model, the effectiveness and convergence of the difference scheme are numerically verified. Furthermore, the impact of some key parameters on the stock loan value and optimal redemption price have been discussed in detail. Finally, we draw conclusions in Section 5.

2. Mathematical model

In this section, we first present the space-fractional differential equation governing stock loan values with finite maturity. Then, financially, the corresponding time-varying moving boundary and terminal conditions will be given to close the pricing system.

In the FMLS model, the log value of stock $x_t = \ln S_t$, under the risk neutral measure $\mathbb{Q}$, follows a stochastic differential equation of the maximally skewed Lévy process [10]:

$$dx_t = (r - D - \nu)dt + \sigma dL_t^{\alpha,-1}, \tag{1}$$

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