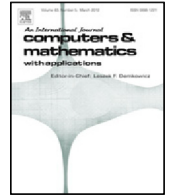




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The fibering map approach to a nonlocal problem involving $p(x)$ -Laplacian[☆]

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ABSTRACT

We study the existence of positive solutions for a $p(x)$ -Kirchhoff problem. The main tool used is the fibering map approach for the corresponding Nehari manifold.

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1. Introduction

The study of differential equations and variational problems with nonstandard $p(x)$ -growth conditions has been a new and interesting topic. Such problems arise from the study of electrorheological fluids (see Ružička [1]), and elastic mechanics (see Zhikov [2]). It also has wide applications in different research fields, such as image processing model (see e.g., [3,4]), stationary thermorheological viscous flows (see [5]) and the mathematical description of the processes filtration of an idea barotropic gas through a porous medium (see [6]).

In recent years, many problems on $p(x)$ -Laplacian type have been studied by many authors using various methods, for example, variational method (see, e.g., [7–16]), topological method (see, e.g., [17,18]), subsupersolution method (see, e.g., [19]), Nehari manifold method (see, e.g., [20]) and monotone mapping theory (see, e.g., [21]). The fibering map approach for description of the Nehari manifolds and seeking solutions in an appropriate subset of the Sobolev space is introduced by Drabek and Pohozaev in [22] and is also discussed by Brown and Zhang in [23]. Several papers are devoted to this method, we just quote a few, for example, see [24,25].

In [26], the authors have considered the following $p(x)$ -Kirchhoff-type problem:

$$\begin{cases} -M\left(\int_{\Omega} \frac{1}{p(x)} |\nabla u|^{p(x)} dx\right) \Delta_{p(x)} u = \lambda(a(x)|u|^{q(x)-2}u + b(x)|u|^{r(x)-2}u), & \text{in } \Omega, \\ u = 0, & \text{on } \partial\Omega, \end{cases} \quad (P)$$

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where $-\Delta_{p(x)}u = -\operatorname{div}(|\nabla u|^{p(x)-2}\nabla u)$ is the $p(x)$ -Laplacian operator, Ω is a smooth boundary domain in $\mathbb{R}^N$, $M(t)$ is a continuous function, and $p, q, r \in C(\overline{\Omega})$, $q^- := \inf_{x \in \overline{\Omega}} q(x) > 1$, $r^- := \inf_{x \in \overline{\Omega}} r(x) > 1$, $a(x), b(x) > 0$ for $x \in \overline{\Omega}$.

For the case when $a \in L^{\alpha(x)}(\Omega)$, $\alpha(x) = \frac{p(x)}{p(x)-q(x)}$ and $b \in L^{\gamma(x)}(\Omega)$, $\gamma(x) = \frac{p^*(x)}{p^*(x)-r(x)}$, with $p^*(x) = \frac{Np(x)}{N-p(x)}$ if $p(x) < N$; $p^*(x) = +\infty$ if $p(x) \geq N$, and $1 < q^- \leq q^+ < \nu p^-$, $\max\{\mu p^+, \frac{p^+}{1-d}\} < r^- \leq r^+ < p^*(x)$ and $p^+ < N$, the authors in [26] have proved the existence of positive constant λ^* such that any $\lambda \in (0, \lambda^*)$, problem (P) possesses a nontrivial solution.

Such nonlocal elliptic problem like (P) has received a lot of attention and some important and interesting results have been established in, for example, [27–31]. Inspired by the above facts and aforementioned papers, the main purpose of this paper is to study the existence of two positive solutions for problem (P). Our tool used here is the fibering map approach, which is also used by Brown and Wu [32]. Before stating our main results, we first make some assumptions on the functions M, a, b and q, r . Hereafter, $M \in C^1([0, +\infty))$ is supposed to verify the following assumption:

(H_M) (1) There exist $m_1 \geq m_0 > 0$ and $\mu \geq \nu > 2$ such that

$$m_0 t^{\nu-2} \leq M'(t) \leq m_1 t^{\mu-2};$$

(2) There exists $d \in \mathbb{R}$ with $0 < d < 1$, such that

$$tM'(t) \leq \frac{d}{1-d}M(t).$$

Note that (H_M)(2) implies that $\widehat{M}(t) \geq (1-d)M(t)t$, where $\widehat{M}(t) = \int_0^t M(s)ds$.

For the potential a, b , we make the following assumption:

(H_{a,b}) $0 < a(x) \in L^{\alpha(x)}(\Omega)$ with $1 < \alpha(x) \in C(\overline{\Omega})$ and $0 < b(x) \in L^{\gamma(x)}(\Omega)$ with $1 < \gamma(x) \in C(\overline{\Omega})$.

For the functions q, r , we suppose it satisfies the following condition in this paper.

(H_{q,r}) $1 < q(x) \leq q^+ < \min\{\frac{p^*(x)}{\alpha'(x)}, \nu p^-\}$ and $\max\{\frac{p^+}{1-d}, \mu p^+\} < r^- \leq r(x) < \frac{p^*(x)}{\gamma'(x)}$, where $\alpha'(x) = \frac{\alpha(x)}{\alpha(x)-1}$, $\gamma'(x) = \frac{\gamma(x)}{\gamma(x)-1}$.

We are now in the position to state our main results.

Theorem 1. Suppose that (H_M), (H_{a,b}) and (H_{r,q}) hold. Then there exists a $\lambda_0 > 0$ such that for any $\lambda \in (0, \lambda_0)$, problem (P) possesses at least two positive solutions.

Now, we present the following example to illustrate Theorem 1.

Example 1.1. Let $N = 4$, $\mu = \nu = 3$, $m_0 = m_1 = 1$ and $d = \frac{2}{3}$. We consider the following problem

$$\begin{cases} -\frac{1}{2} \left(\int_{\Omega} \frac{1}{3} |\nabla u|^3 dx \right)^2 \Delta_3 u = \lambda(u + |u|^8 u), & \text{in } \Omega, \\ u = 0, & \text{on } \partial\Omega, \end{cases}$$

where $a(x) = b(x) \equiv 1$, $q(x) = 2$, $p(x) = 3$, $r(x) = 10$ and $M(t) = \frac{t^2}{2}$ for $t \in [0, +\infty)$.

Proof. We clearly have $M \in C^1([0, +\infty))$ and $M'(t) = t$, $\widehat{M}(t) = \frac{1}{6}t^2$. So condition (H_M) has been satisfied. Obviously, $a \in L^{\alpha(x)}(\Omega)$ and $b \in L^{\gamma(x)}(\Omega)$, where $\alpha(x) = \gamma(x) = 8$, so condition (H_{a,b}) holds. Direct calculations show $\alpha'(x) = \gamma'(x) = \frac{8}{7}$ and $p^*(x) = \frac{Np(x)}{N-p(x)} = 12$. Therefore, owing to our assumptions, we have

$$1 < q(x) = 2 < \min\{\frac{p^*(x)}{\alpha'(x)}, \nu p^-\} = \min\{10.5, 9\}$$

and

$$9 = \max\{\frac{p^+}{1-d}, \mu p^+\} < 10 < \frac{p^*(x)}{\gamma'(x)} = 10.5.$$

So condition (H_{r,q}) has been satisfied. Therefore, all the assumptions of Theorem 1 have been verified, which completes the proof. $\square$

The rest of this paper is organized as follows. In Section 2, we recall some necessary preliminaries, which will be used in our investigation in Section 3. In Section 3, we prove the main results of the paper.

2. Preliminary

In order to discuss problem (P), we need some theories on $W_0^{k,p(x)}(\Omega)$ which we will call variable exponent Sobolev space. For more details on the basic properties of these spaces, we refer the readers to Fan and Zhao [33], Kovacik, Rakosnik [34], Fan and Zhang [35].

Set

$$C_+(\overline{\Omega}) = \{h \in C(\overline{\Omega}) : h(x) > 1 \text{ for any } x \in \overline{\Omega}\}.$$

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