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BCR method for solving generalized coupled Sylvester equations over centrosymmetric or anti-centrosymmetric matrix[☆]

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ABSTRACT

This paper introduces another version of biconjugate residual method (BCR) for solving the generalized coupled Sylvester matrix equations over centrosymmetric or anti-centrosymmetric matrix. We prove this version of BCR algorithm can find the centrosymmetric solution group of the generalized coupled matrix equations for any initial matrix group within finite steps in the absence of round-off errors. Furthermore, a method is provided for choosing the initial matrices to obtain the least norm solution of the problem. At last, some numerical examples are provided to illustrate the efficiency and validity of methods we have proposed.

1. Introduction

Throughout this paper, the notion $R^{m\times n}$ denote the set of $m \times n$ real matrices, $A \in R^{m\times n}$ will denote a real $m \times n$ real matrix, R^n will denote n dimensional vector space, $e_i \in R^n$ will denote n dimensional column unit vector with ith entry 1. A^T will denote the transpose of the matrix. $\mathscr{R}(A)$ will signify the column space of matrix A. An $m \times n$ matrix of zeros will be denoted by $\boldsymbol{O}^{m\times n}$ and an $m \times n$ matrix with all entries one will be denoted by $\mathbf{1}^{m\times n}$. For any A, B, $A \otimes B$ will denote the Kronecker product $A \otimes B = (a_{ij}B)$. When $A, B \in R^{m\times n}$, their inner product is defined $\langle A, B\rangle = tr(B^TA)$. The notion $tr(A)$ will denote the trace of matrix A. Let $X = (x_1, x_2, \ldots, x_n) \in R^{m\times n}$, we use $vec(X)$ denote an mn column vector $(x_1^T, x_2^T, \ldots, x_n^T)^T$.

The matrix $S \in R^{n\times n}$ is said quasi-identity matrix if $S = [e_n, e_{n-1}, \ldots, e_1]$, where e_i is unit vector with ith entry 1.

Definition 1.1. The matrix $X \in R^{m\times m}$ is said centrosymmetric matrix if $SXS = X$, denote $X \in CSR^{m\times m}(S)$, where S is n-order quasi-identity matrix.

Definition 1.2. The matrix $X \in R^{m\times m}$ is said anti-centrosymmetric matrix if $SXS = -X$, denote $X \in CASR^{m\times m}(S)$, where S is n-order quasi-identity matrix.

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Matrix equation has extensive applications in many fields such as system theory, control theory, stability analysis, signal image processing and photogrammetry [1–9]. Centrosymmetric matrices have been widely discussed, which are very useful in engineering problems, information theory, linear system theory, linear estimation theory and numerical analysis theory, and others [10–15]. How to solve this matrix equation has become a subject of widespread concern [16–19]. There are many matrix equations and many methods for solving those matrix equations been discussed in [20–76]. QR-fractoring methods are proposed in [20,21]. Matrix splitting and gradient method are used widely [22–28]. Peng [29], Huang [30] and Khorsand Zak [31] present an iterative method for solving the system $AXB = C$. Navarra et al. investigated general solution of $A_1XB_1 = C_1, A_2XB_2 = C_2$ [32]. Piao et al. studied the necessary and sufficient conditions for the existence of the solution and the expressions of the matrix equation $AX + X^TC = B$ [33]. Ding et al. investigated the equation $AXB + CYD = F$ with extending Jacobi and Gauss–Seidel iteration on linear equation [34]. Recently, many Krylov subspace methods of linear system are extended to solve linear matrix equation. Liang and Liu proposed a modified conjugate gradient method to solve the following problem $A_1XB_1 + C_1X^TD_1 = F_1, A_2XB_2 + C_2X^TD_2 = F_2$ [35]. Dehghan and Hajarian studied the generalized coupled Sylvester matrix equations $AXB + CYD = M, EXF + GYH = N$ and presented a modified conjugate gradient method to solve these matrix equations over the generalized centrosymmetric matrix pair (X, Y) [36]. Xie and Ma also gave an iterative algorithm to solve the system over reflexive or anti-reflexive matrix $AXB + CY^TD = S_1, EX^TF + GYH = S_2$ [37]. Huang and Ma proposed a modified conjugate gradient method to solve generalized coupled Sylvester-transpose matrix equations [38]. Dehghan and Hajarian constructed an iterative method to solve the general coupled matrix equations over generalized bisymmetric matrix [39,40] and an iterative algorithm for the reflexive solutions of the generalized coupled Sylvester matrix equations [41]. The matrix forms of CGS, GPBiCG, QMRCGSTAB, BiCOR, Bi-CGSTAB, CORS, Bi-CG and Bi-CR methods were given to solve linear matrix equations [42–46]. In [47] Hajarian solves the following generalized coupled Sylvester conjugate matrix equation via a HS version of BCR algorithm

$$\begin{cases} A_1XB_1 + C_1YD_1 + E_1X^TF_1 = G_1, \\ A_2XB_2 + C_2YD_2 + E_2Y^TF_2 = G_2 \end{cases} \tag{1.1}$$

and in [48] also solve the following generalized Sylvester matrix equation over the generalized reflexive matrices X and Y

$$\sum_{i=1}^{f} A_iXB_i + \sum_{j=1}^{g} C_jYD_j = E. \tag{1.2}$$

In this paper, we consider the generalized coupled Sylvester matrix equation

$$\sum_{j=1}^{n} A_{ij}X_jB_{ij} = E_i, \ \ i = 1, 2, \dots, m, \tag{1.3}$$

where $A_{ij} \in R^{s_i\times r_j}$, $B_{ij} \in R^{r_j\times t_i}$, $E_i \in R^{m_i\times n_i}$ and $X_j \in R^{r_j\times r_j}$. We will solve the system (1.3) over centrosymmetric or anti-centrosymmetric matrix with another HS version of BCR algorithm and proved the iterative solution can be obtained within finite steps ignoring roundoff error, then we show that the least Frobenius norm solution can be determined. At last, we give some numerical examples to illustrate the efficiency.

The remainder of this paper is organized as follows. In Section 2, we give a BCR method for solving matrix equation system (1.3) and prove that a solution matrix group $\{X_j^*\}$ of (1.3) can be obtained within a finite number of iterative steps in the absence of round-off error for any initial value. In Section 3, we show that the minimum-norm solution can be obtained by choosing a special kind of initial matrix. In Section 4, we present some numerical experiments. Finally, we give our conclusions in Section 5.

2. Biconjugate residual method

In this section, we will give a new biconjugate residual method to solve the matrix equations (1.3). Q'Leary proposed the Block-CG algorithm for solving linear system. M. Vespucci and C. Broyden [49] gave some different version of BCR from general framework of Block-CG algorithm, they showed the BCR2A-b is more efficient than the others vision in numerical experiments. In this section, we will introduce another biconjugate residual method, we call it BCR2A, to solve the matrix equations (1.3), which is different from the version in [47,48]. In this algorithm, we use trace of a matrix as its inner product. If $A, B \in R^{m\times n}$, their inner product can be defined $\langle A, B\rangle = tr(A^TB)$. The Frobenius norm of a matrix A is square root of $tr(A^TA)$, denoted $\|A\|$. According to the proprieties of trace of matrix, we know $tr(A^T) = tr(A)$ and if A, B is square matrix, then $tr(AB) = tr(BA)$.

Lemma 2.1 ([37]). *Let $A, B \in R^{p\times m}$, then*

$$\langle A, B\rangle = \mathrm{tr}(B^TA) = \mathrm{tr}(AB^T) = \mathrm{tr}(A^TB) = \langle B, A\rangle. \tag{2.1}$$

By Lemma 2.1, we can have

$$\langle Y, AXB\rangle = \langle X, A^TYB^T\rangle. \tag{2.2}$$