



Contents lists available at ScienceDirect

Computers and Mathematics with Applications

journal homepage: www.elsevier.com/locate/camwa

Global existence of three-dimensional incompressible magneto-micropolar system with mixed partial dissipation, magnetic diffusion and angular viscosity

Liangliang Ma *

Department of Mathematics, Sichuan University, Chengdu 610064, PR China

Department of Mathematics and Computer, Panzhihua University, Panzhihua 617000, PR China

ARTICLE INFO

Article history:

Received 12 April 2017

Received in revised form 27 July 2017

Accepted 4 September 2017

Available online xxxx

Keywords:

Magneto-micropolar fluid

Smooth solution

Mixed partial viscosity

Global existence

Energy method

ABSTRACT

The magneto-micropolar fluid flows describe the motion of electrically conducting micropolar fluids in the presence of a magnetic field. The issue of whether the strong solution of magneto-micropolar equations in three-dimensional can exist globally in time with large initial data is still unknown. In this paper, we deal with the Cauchy problem of the three-dimensional magneto-micropolar system with mixed partial dissipation, magnetic diffusion and angular viscosity. More precisely, the global existence of smooth solutions to the three-dimensional incompressible magneto-micropolar fluid equations with mixed partial dissipation, magnetic diffusion and angular viscosity are obtained by energy method under the assumption that H^1 -norm of the initial data (u_0, b_0, w_0) sufficiently small, namely $\|u_0, b_0, w_0\|_{H^1(\mathbb{R}^3)}^2 \leq \frac{\varepsilon}{2}$, where ε is a sufficiently small positive number. This work follows the techniques in the paper of Cao and Wu (2011).

© 2017 Elsevier Ltd. All rights reserved.

1. Introduction and main results

This paper aims at the global existence of smooth solutions to the three-dimensional incompressible magneto-micropolar fluid equations with mixed partial dissipation, magnetic diffusion and angular viscosity. The standard three-dimensional incompressible magneto-micropolar system can be written as follows

$$\begin{cases} \partial_t u + (u \cdot \nabla)u + \nabla(p + \frac{1}{2}|b|^2) - (b \cdot \nabla)b - 2\chi \nabla \times w = (\mu + \chi)\Delta u, \\ \partial_t b + (u \cdot \nabla)b = \nu \Delta b + (b \cdot \nabla)u, \\ \partial_t w + (u \cdot \nabla)w - \kappa \nabla \operatorname{div} w + 2\chi w - 2\chi \nabla \times u = \gamma \Delta w, \\ \nabla \cdot u = \nabla \cdot b = 0, \end{cases} \quad (1.1)$$

where $(x, y, z) \in \mathbb{R}^3$ and $t \geq 0$, $u = (u_1(x, y, z, t), u_2(x, y, z, t), u_3(x, y, z, t))$ denotes the velocity field in $\mathbb{R}^3 \times \mathbb{R}^+$, $b = (b_1(x, y, z, t), b_2(x, y, z, t), b_3(x, y, z, t))$ denotes the magnetic field in $\mathbb{R}^3 \times \mathbb{R}^+$, $w = (w_1(x, y, z, t), w_2(x, y, z, t), w_3(x, y, z, t))$ denotes the micro-rotational velocity field at point $(x, y, z) \in \mathbb{R}^3$ at time $t > 0$, and $p(x, y, z, t)$ denotes the scalar pressure. The non-negative constants $\mu, \chi, \frac{1}{\nu}$ are the coefficients of kinematic viscosity, vortex viscosity, magnetic Reynolds number, and κ, γ are angular viscosities.

The magneto-micropolar fluid equations are not only important in engineering and physics, but also mathematically significant. The mathematical study of the magneto-micropolar fluid equations started in the seventies and has been continued

* Correspondence to: Department of Mathematics and Computer, Panzhihua University, Panzhihua 617000, PR China

E-mail address: mlpzh@126.com.

by many authors (see, e.g., [1–3]). The three-dimensional magneto-micropolar fluid equations describe a micropolar fluid moving into a magnetic field. Micropolar fluids represent a class of fluids with nonsymmetric stress tensor (see, e.g., [4–6]). In this paper, we consider the following three-dimensional incompressible magneto-micropolar system

$$\begin{cases} \partial_t u + (u \cdot \nabla)u + \nabla(p + \frac{1}{2}|b|^2) - (b \cdot \nabla)b - 2\chi \nabla \times w = \mu_1 \partial_{xx} u + \mu_2 \partial_{yy} u + \mu_3 \partial_{zz} u + \chi \Delta u, \\ \partial_t b + (u \cdot \nabla)b = \nu_1 \partial_{xx} b + \nu_2 \partial_{yy} b + \nu_3 \partial_{zz} b + (b \cdot \nabla)u, \\ \partial_t w + (u \cdot \nabla)w - \kappa \nabla \operatorname{div} w + 2\chi w - 2\chi \nabla \times u = \gamma_1 \partial_{xx} w + \gamma_2 \partial_{yy} w + \gamma_3 \partial_{zz} w, \\ \nabla \cdot u = \nabla \cdot b = 0. \end{cases} \quad (1.2)$$

Clearly, if $\mu_1 = \mu_2 = \mu_3 = \mu$, $\nu_1 = \nu_2 = \nu_3 = \nu$, $\gamma_1 = \gamma_2 = \gamma_3 = \gamma$, then (1.2) reduces to the standard three-dimensional magneto-micropolar fluid equations in (1.1). Furthermore, (1.2) allows us to explore the smooth effects of various mixed partial dissipation, magnetic diffusion and angular viscosity.

It is well known that the global regularity or finite time singularity of the classical solution for the three-dimensional Navier–Stokes and magnetohydrodynamic (MHD) equations are still challenge open problems. The same problem for the three-dimensional magneto-micropolar system is also difficult. Yuan studied the regularity of weak solutions and the blow-up criteria for smooth solutions to the magneto-micropolar fluid equations in \mathbb{R}^3 in [7]. The global existence of strong solutions with small initial data was shown in [8]. Li and Chen considered the regularity criteria of the weak solutions for the 3D magneto-micropolar fluid equations by energy method and Littlewood–Paley decomposition, and proved some regularity criteria involving the pressure or pressure gradient for weak solutions in Lebesgue, Lorentz, BMO and Besov spaces in [9]. Rojas-Medar proved the local existence and uniqueness of the strong solution in three dimensions for any initial data in [10]. Rojas-Medar and Boldrini also obtained the local existence of weak solutions for three dimensional magneto-micropolar in [11]. Gala established the regularity criteria for the 3D magneto-micropolar fluid equations in Morrey–Campanato spaces in [12]. Wang and Wang investigated the initial value problem and established global existence of smooth solutions for the three-dimensional magneto-micropolar fluid equations with mixed partial viscosity in [13] following the work of Cao and Wu [14].

For two-dimensional case, Regmi and Wu studied the global existence and regularity of classical solutions to the 2D incompressible magneto-micropolar equations with partial dissipation in [5]. Yamazaki studied the two-dimensional magneto-micropolar fluid system and showed that with zero angular viscosity the solution triple remains smooth for all times in [15]. Wang and Wang derived a blow-up criterion of smooth solutions to the incompressible magneto-micropolar fluid equations with partial viscosity in [16]. There is one more reference [17] on the magneto-micropolar fluid equations, this paper obtained the global regularity for the 2D fractional magneto-micropolar equations with various fractional dissipation.

If the magnetic field $b = 0$, then Eqs. (1.1) and (1.2) become the micropolar fluid equations. Dong and Zhang [18] studied the global regularity of smooth solution of the 2D incompressible micropolar fluid flows with zero angular viscosity. Łukaszewicz [19] established the global existence of weak solutions for 3D micropolar fluid equations. Chen [20] obtained the global well-posedness of the 2D incompressible micropolar fluid flows with mixed partial viscosity and angular viscosity. Dong–Wu considered the global well-posedness and large-time decay for the 2D micropolar equations in [21]. If $w = 0$ and $\chi = 0$, Eqs. (1.1) and (1.2) reduce to the magnetohydrodynamic(MHD) equations, which were studied extensively in [14,22–24], and the references therein. Furthermore, if $b = w = 0$ and $\chi = 0$, Eqs. (1.1) and (1.2) become the incompressible Navier–Stokes equations, which were studied by many scholars too, such as [25–27] and the references therein.

For the sake of simplicity, let us make these assumptions in the following. Denoting

$$A^T = \begin{pmatrix} \mu_1 \\ \mu_2 \\ \mu_3 \end{pmatrix}, \quad B^T = \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}, \quad C^T = \begin{pmatrix} \gamma_1 \\ \gamma_2 \\ \gamma_3 \end{pmatrix}$$

as the matrix of kinematic viscous coefficients, the matrix of magnetic diffusion coefficients and the matrix of angular viscous coefficients.

We supplement the system (1.2) with the initial data

$$u(x, y, z, 0) = u_0(x, y, z), \quad b(x, y, z, 0) = b_0(x, y, z), \quad \text{and} \quad w(x, y, z, 0) = w_0(x, y, z) \quad (1.3)$$

with divergence-free conditions $\operatorname{div} u_0 = 0$ and $\operatorname{div} b_0 = 0$.

As the recent work [13] on this topic, Wang and Wang established the global well-posedness for three-dimensional magneto-micropolar fluid equations with mixed partial viscosity with small initial data. More precisely, they showed the global existence of smooth solutions to the Cauchy problem (1.2)–(1.3) with

$$\text{Case 1. } (A | B | C) = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{pmatrix}, \quad \text{Case 2. } (A | B | C) = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix},$$

Download English Version:

<https://daneshyari.com/en/article/6892255>

Download Persian Version:

<https://daneshyari.com/article/6892255>

[Daneshyari.com](https://daneshyari.com)