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Computable majorants of the limit load in Hencky's plasticity problems

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ABSTRACT

We propose a new method for analyzing the limit (safe) load of elastoplastic media governed by the Hencky plasticity law and deduce fully computable bounds of this load. The main idea of the method is based on a combination of kinematic approach and new estimates of the distance to the set of divergence free fields. We show that two sided bounds of the limit load are sharp and the computational efficiency of the method is confirmed by numerical experiments.

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1. Introduction

The presence of a limit load is a feature of some elastoplastic problems subject to proportional loading of the form $L_0 + \lambda L$. Here, L_0 and L denote fixed linear functionals generated by external loads and $\lambda \geq 0$ is the amplifying load parameter. The limit load is represented by a limit value λ^* of λ beyond which the problem has no solution (classical or generalized). Physically, this value corresponds to the situation where the external forces cannot be balanced by internal stresses and, therefore, the body must collapse. The limit value is trivially equal to $+\infty$ in elasticity or plasticity with unbounded hardening. However, in perfect plasticity or plasticity with limited hardening, the value λ^* may be finite [1–6]. The knowledge of λ^* is an important pre-condition for correctness (solvability) of elasto-plastic problems. Furthermore, the knowledge of plastic collapse states is useful in many nonlinear models describing continuous media with limited resistance properties (e.g., bearing capacity of strip-footing or slope stability in soil mechanics [1,3,4,7]).

For an associative perfectly plastic model, it is well known that the limit load and the collapse state can be defined by a specific variational problem, the so-called limit analysis problem (see, e.g., [2,5]). This problem can be formulated either in terms of stresses (the static principle) or in terms of displacements (the kinematic principle). The corresponding mixed formulation is analyzed in [2,8]. It is important to note that the quasistatic model and its simplified static version (parametrized Hencky's model) lead to the same limit analysis problem. Limit analysis for some nonassociative models or models with limited hardening can be defined by the theory of bipotentials [3,6]. Limit analysis is also closely related to more complex problems like shakedown analysis [9] or load capacity [10,11].

We focus on the Hencky plasticity problem [5,12,13], the kinematic principle and the choice $L_0 = 0$, for the sake of simplicity. Then the limit analysis problem contains the load isoperimetric constraint $L(w) = 1$ and other equality or

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inequality constraints depending on a prescribed yield criterion. The problem can be solved by the augmented Lagrangian method [10] or by second-order cone programming [14]. Another approach based on penalization and the indirect method of incremental limit analysis has been recently developed in [15–19]. To estimate λ^* from below within the kinematic approach, one can use the so-called truncation method [18,19] in which unbounded yield surfaces defining the constitutive model are approximated by bounded ones. This technique also significantly reduces locking phenomena often observed in computational plasticity.

Typically, bounds of λ^* found by computational methods are not fully guaranteed due to replacing a differential problem by a discrete one and various errors arising in computational procedures used to solve the discrete problem. For some specific geometries and yield criteria, guaranteed bounds of λ^* follow from analytical methods [1,3,19], but these are rather special cases more related to theoretical analysis of the limit load phenomenon. Our goal is to develop a unified approach of computing guaranteed upper bounds of λ^* which could be applied to a wide spectrum of problems with different geometries and different yield laws. The principal intention is that bounds should be guaranteed even if they are computed by the help of finite dimensional approximations. The first step on this way has been made in [18], where such a method was developed for bounded yield surfaces.

In this paper, we present an idea how to provide computable and fully guaranteed upper bounds of limit loads for problems with unbounded yield surfaces. Our method is based on estimates of the distance between a vector valued function in the space $W^{1,2}(\Omega, \mathbb{R}^d)$ and a subspace of this set that arises due to the constraints in the variational problem of the kinematic approach to limit load analysis. In the case of the classical Hencky problem with the von Mises yield law, the constraints lead to the divergence free condition.

Estimates of the distance between a function from $W^{1,2}(\Omega, \mathbb{R}^d)$ and the space $\mathbb{S}$ of divergence free (solenoidal) functions follow from the inf–sup (LBB) condition widely used (in different forms) in the theory of incompressible media (see [20–24]). For some classes of domains and functions satisfying the Dirichlet boundary conditions, estimates of the LBB constant can be found by analytical methods (see [25–27] and the references therein).

Estimates of the distance to the set of divergence free fields contain these constants. They have been derived in [28,29] and some other publications related to a posteriori estimates for viscous flow problems. In the recent papers [30–32], advanced forms of the estimates adapted to domain decompositions and mixed boundary conditions are presented. We use these results in our analysis of the limit load.

The paper is organized as follows. Section 2 presents some preliminaries related to the Hencky plasticity problem and definitions of limit load parameters. A particular attention is paid to the von Mises yield criterion. In Section 3, we discuss estimates of the distance to the space of divergence free displacement fields and use them to derive a guaranteed and fully computable upper bound of the limit load. Section 4 deals with discrete problems generated by conforming finite element approximations of the plasticity problem. They are subsequently used in computations together with the indirect incremental method of limit analysis. Section 5 is devoted to numerical examples. It illustrates the efficiency of computed majorants of λ^* in several model examples. The last Section 6 contains concluding remarks.

2. Hencky plasticity problem

In this section, we introduce the Hencky plasticity problem and the corresponding limit load. First, we formulate the limit analysis problem for an abstract yield criterion in order to summarize and extend some results from [18] dealing with the guaranteed upper bounds of the limit load for bounded yield surfaces. This is done in order to present the main idea of our approach in the most general form. Later, the results are applied to elastoplastic problems with von Mises yield condition and this classical model is considered in the remaining part of the paper.

2.1. The variational problem

Let Ω be a bounded domain in $\mathbb{R}^d$, $d = 2, 3$, with the Lipschitz continuous boundary $\partial\Omega$ and Γ_D, Γ_f be open parts of $\partial\Omega$ such that

$$\text{meas}_{d-1}\Gamma_D > 0, \quad \Gamma_D \cap \Gamma_f = \emptyset, \quad \bar{\Gamma}_D \cup \bar{\Gamma}_f = \partial\Omega.$$

We assume that the body is fixed on Γ_D. Next, let $f \in L^2(\Gamma_f; \mathbb{R}^d)$, $F \in L^2(\Omega; \mathbb{R}^d)$ denote the surface and volume forces, respectively. Then,

$$L(v) = \int_\Omega F \cdot v \, dx + \int_{\Gamma_f} f \cdot v \, ds, \quad v \in \mathbb{V}, \tag{2.1}$$

is the load functional defined on the space of admissible displacements:

$$\mathbb{V} := \{v \in W^{1,2}(\Omega; \mathbb{R}^d) \mid v = 0 \text{ on } \Gamma_D\}.$$

By $\mathbb{M}^{d\times d}_{sym}$, we denote the space of symmetric $d \times d$ matrices supplied with the scalar product $e : \eta := e_{ij}\eta_{ij}$ and the norm $|e|^2 = e : e$. In order to introduce the constitutive law for the generalized Hencky problem, we define a closed, convex set $B \subset \mathbb{M}^{d\times d}_{sym}$ containing a vicinity of the origin. This set represents the stresses admissible in accordance with the accepted law

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