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Existence result for fractional Schrödinger–Poisson systems involving a Bessel operator without Ambrosetti–Rabinowitz condition

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ABSTRACT

The present study is concerned with the nontrivial solutions for fractional Schrödinger–Poisson system with the Bessel operator. Under certain assumptions on the nonlinearity f, a nontrivial nonnegative solution is obtained by perturbation method for the given problem. In particular, the Ambrosetti–Rabinowitz type condition or the monotone assumption on the nonlinearity is unnecessary.

1. Introduction and main results

In this paper, we are concerned with the existence of nontrivial solutions for the following fractional Schrödinger–Poisson systems:

$$\begin{cases}(I-\Delta)^s u+\phi u=f(x,u), & x\in\mathbb{R}^3,\\ (-\Delta)^t\phi=u^2, & x\in\mathbb{R}^3,\end{cases}\tag{1.1}$$

where $(I-\Delta)^s$ is the Bessel operator and $(-\Delta)^t$ is a fractional Laplacian operator for $s\in(3/4,1)$ and $t\in(0,1)$, respectively. The nonlinear term $f(x,u)$ is assumed to verify the following conditions:

(f_1) *$f\in C(\mathbb{R}^3\times\mathbb{R},\mathbb{R})$ and there exist constants $C_0>0$ and $4<p<2_s^*=6/(3-2s)$ such that*

$$|f(x,t)|\le C_0(1+|u|^{p-1})\ \text{ for any }\ (x,t)\in\mathbb{R}^3\times\mathbb{R};$$

(f_2) *$f(x,0)=0$ and $f(x,t)=o(t)$ uniformly in $x\in\mathbb{R}^3$ as $t\to0$;*
(f_3) *$\lim_{t\to+\infty}\frac{F(x,t)}{t^4}=+\infty$ uniformly in $x\in\mathbb{R}^3$, where $F(x,t)=\int_0^t f(x,s)ds$;*
(f_4) $H(x,t)\triangleq\frac{1}{4}tf(x,t)-F(x,t)\ge0$ for any $(x,t)\in\mathbb{R}^3\times\mathbb{R}$.

If $\phi=0$ in (1.1), it turns into an equation involving the nonlocal operator $(I-\Delta)^s$ arising in the study of standing waves $\psi=\psi(t,x)$ for the Schrödinger–Klein–Gordon equations of the form

$$i\frac{\partial\psi}{\partial t}=(I-\Delta)^\alpha\psi-f(x,\psi),\ \ (t,x)\in\mathbb{R}\times\mathbb{R}^N$$

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which describe the behavior of bosons. For a physical introduction to these fractional equations, the reader can refer to [1,2] and their references therein.

Taking $s = t = 1$ into account, then the system (1.1) can reduce to be the following Schrödinger–Poisson system:

$$\begin{cases} -\Delta u + u + \phi u = f(x,u), & x \in \mathbb{R}^3, \\ -\Delta \phi = u^2, & x \in \mathbb{R}^3 \end{cases}$$

proposed by Benci–Fortunato [3,4] to represent solitary waves for nonlinear Schördinger type equations and look for the existence of standing waves interacting with an unknown electrostatic field. We refer the reader to [5–11] for some related and important results.

The nonlinear fractional Schrödinger–Poisson systems like (1.1) come from the following fractional Schrödinger equation

$$(-\Delta)^s u + V(x)u = f(x,u), \quad x \in \mathbb{R}^N \tag{1.2}$$

used to study the standing wave solutions $\psi(x,t) = u(x)e^{-i\omega t}$ for the equation

$$i\hbar \frac{\partial \psi}{\partial t} = \hbar^2(-\Delta)^\alpha \psi + W(x)\psi - f(x,\psi), \quad x \in \mathbb{R}^N,$$

where $\hbar$ is the Planck's constant, $W : \mathbb{R}^N \to \mathbb{R}$ is an external potential and f a suitable nonlinearity. Since the fractional Schrödinger equation appears in problems involving nonlinear optics, plasma physics and condensed matter physics, it is one of the main objects of the fractional quantum mechanic. To know more about the study of fractional Schrödinger equations, the reader can refer to [1,2,12–18] and their references therein.

However the research of the fractional Schrördinger–Poisson system is not so fruitful, please see [19–23] for example. Meanwhile to the best knowledge of us, there are few results on the Schrödinger equation involving a Bessel operator. On the other hand, because of the lack of scaling properties (there is no standard group action under which $(I - \Delta)^\alpha$ behaves as a local differential operator) for the Bessel operator, the study of problem (1.1) does not seem to be trivial. The Bessel operator $(I - \Delta)^s$ with $0 < s < 1$ is related to the pseudo-relativistic Schördinger operator $(m^2 - \Delta)^{1/2} - m$ $(m > 0)$ and recently a lot of attention is paid to equations involving it, see [24,25] for example. S. Secchi [26] proved the existence and multiple existence results for the following Schördinger equation

$$(I - \Delta)^s u = \mu b(x)|u|^{p-2}u + c(x)|u|^{q-2}u \ \text{ in } \mathbb{R}^N,$$

where $\mu > 0$ is a parameter, $s \in (0,1)$ and $p, q \in (1, 2_s^*)$. Also there has some meaningful research on Schördinger equation involving the Bessel operator in [27–29]. Inspired by all the works described above, we try to get the existence of nontrivial nonnegative solution to the system (1.1).

Our main result is as follows:

Theorem 1.1. *Assume (f_1)–(f_4), $s \in (3/4, 1)$ and $t \in (0, 1)$, then the system (1.1) admits at least a nontrivial nonnegative solution.*

Remark 1.2.

(1) The choice of $s > 3/4$ ensures that the interval $(4, 2_s^*)$ in (f_1) is non-degenerated.
(2) It is well-known that the assumption (f_4) is weaker than the Ambrosetti–Rabinowitz type condition:

$$(AR) \ \text{there exists } \mu > 4 \text{ such that } tf(x,t) - \mu F(x,t) \geq 0 \ \text{ for any } (x,t) \in \mathbb{R}^3 \times \mathbb{R},$$

and the monotone assumption: for any $x \in \mathbb{R}^3$

$$(M) \ \frac{f(x,t)}{t^3} \ \text{is strictly increasing on } t \in (0, +\infty) \text{ and strictly decreasing on } t \in (-\infty, 0).$$

Now we give our main idea for the proof of Theorem 1.1. Since the embedding $L^{s,2}(\mathbb{R}^3)$ (see Section 2) into $L^q(\mathbb{R}^3)$ with $q \in (2, 2_s^*)$ is not compact and neither the nonlinear term $f(x,u)$ verifies the (AR) condition nor the monotone assumption (M), it is difficult for us to show the functional I (see Section 2) satisfies the (PS) condition. Motivated by the works [30,31], we introduce some new analytical skills called perturbation method to overcome this difficult. We will give a detailed explanation of how to prove the existence of a nontrivial nonnegative solution in Section 2 because the function spaces are not established.

The paper is organized as follows. In Section 2, we introduce some work spaces and provide several lemmas. In Section 3, the proof of Theorem 1.1 is completed.

Notation. Throughout this paper we shall denote by C and C_i $(i = 1, 2, \ldots)$ for various positive constants whose exact value may change from lines to lines but are not essential to the analysis of problem. $L^p(\mathbb{R}^3)$ $(1 \leq p \leq +\infty)$ is the usual Lebesgue space with the standard norm $|u|_p$. We use "$\to$" and "$\rightharpoonup$" to denote the strong and weak convergence in the related function