Contents lists available at ScienceDirect

Computers and Mathematics with Applications

journal homepage: www.elsevier.com/locate/camwa

Anisotropic hierarchic solid finite elements for the simulation of passive–active arterial wall models

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ARTICLE INFO

Article history:
Received 20 February 2017
Received in revised form 29 July 2017
Accepted 2 August 2017
Available online xxxx

Keywords:
Anisotropic ansatz
Artery
High-order FEM
Hyperelasticity
Soft tissue

ABSTRACT

A 3D anisotropic hierarchic solid finite element formulation is provided for the passive and active mechanical response of arteries. The artery is modeled as an anisotropic nearly incompressible hyperelastic tube consisting of two layers that correspond to the media and the adventitia. These layers are considered as a fiber-reinforced material consisting of two collagen fiber families that are symmetrically disposed and helically oriented around the tube's axis. The numerical analysis is based on a 3D anisotropic hierarchic solid finite element formulation, including the possibility of varying the polynomial degree in all local directions as well as for all displacement components. For the purpose of verification, analytical solutions are provided for different benchmarks focusing on the aspects of compressible and incompressible 3D stretch, plane strain and plane stress pure shear, as well as a mono-layer anisotropic incompressible circular cylindrical artery of Holzapfel–Gasser–Ogden material under inflation and extension. The resulting solutions, including stress and strain distribution through the arterial wall, are plotted to point out the passive response and different activation levels. In addition the effect of various compressibility levels, including nearly incompressibility, is studied by numerical examples. The robustness and accuracy of the proposed method is demonstrated by comparing the results of different ansatz spaces.

1. Introduction

Several constitutive models have been proposed for the description of the mechanical behavior of soft biological tissues containing collagen fibers. A comprehensive review regarding the mathematical modeling of the mechanical properties of such tissues that constitute the walls of arteries can be found in [1]. A set of three-dimensional constitutive equations was derived from a strain energy density function of exponential type for the arterial wall from data obtained by inflation and longitudinal stretch experiments by [2]. They showed that the thin-walled shell theory can lead to very wrong results under the assumptions of incompressibility and zero residual stresses.

The most commonly used constitutive model for the description of mechanical response of arterial tissue was developed by [3]. The artery was modeled as a thick-walled nonlinear elastic tube consisting of two layers corresponding to the media

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http://dx.doi.org/10.1016/j.camwa.2017.08.007

and adventitia and each layer is treated as a fiber-reinforced material with the fibers corresponding to the collagenous component.

This model has been expanded by including not only the wavy nature of the collagen but also the fraction of both elastin and collagen contained in the media by [4]. They examined this novel strain energy density function by fitting the experimental data from inflation–extension tests performed on rat carotid arteries.

A rate-independent elastoplastic constitutive model for nearly incompressible biological fiber-reinforced composite materials has been proposed by [5]. A key point of the constitutive model is the use of slip systems, which determine the strongly anisotropic elastic and plastic behavior of biological fiber reinforced composites.

A novel five-field Hu–Washizu finite element formulation for nearly inextensible and almost incompressible finite hyperelasticity is developed in [6]. The formulation provides the constraint manifold setting of hyperelasticity with the internal constraints of inextensibility and incompressibility. An exact solution for pure torsion of a circular cylindrical tube with inextensible fibers, is also provided in [6].

Effects of pre-stretch and residual stresses on the mechanical response of arteries have been investigated by considering different reference configurations in [7,8]. A new method to define the mechanical environment of the media and adventitia by avoiding the common prescription of a traction-free reference configuration has been proposed in [9]. By employing a constrained mixture model of the arterial wall, they showed that pre-stretched elastin contributes significantly to both the retraction of arteries, which is observed upon transection, and the opening angle that follows the introduction of a longitudinal cut in an unloaded segment.

Strategies to determine the optimal collagen fiber angle in an iliac artery model has been investigated by [10]. They used three different hypotheses and showed that the optimal fiber angle in the medial layer of human iliac arteries is close to the circumferential direction. Further, the axial prestretch plays an essential role in determining the optimal fiber angle.

Another research work that accounts for residual stresses in arteries has been completed by [8]. In contrast to the usual approach (considering an opening angle of the section of an artery), they analyzed the gradients of suitable stress invariants in thickness direction. Then, utilizing an optimization criterion, they assumed that these gradients have to be smoothed between inner and outer margins of the individual layers.

A new material model is proposed in [11]. It considers the stress-softening observed in cyclic tension tests of arterial walls for applied loads beyond the physiological level. Using continuum damage mechanics and introducing a scalar-valued internal variable, a fiber damage model is presented to consider the microscopic damage in the reinforcing collagen fiber families.

Numerical difficulties associated with satisfying the incompressibility condition in hyperelastic materials have attracted special attention. A modified anisotropic model, considering a volumetric anisotropic contribution, has been presented by [12]. Utilizing three simple deformations (pure dilatation, pure shear, and uniaxial stretch), they showed that unlike the compressible Holzapfel–Gasser–Ogden (HGO-C) model, the proposed modified model can correctly predict the compressible anisotropic response.

Further experimental evidence on the compressibility of arteries (the porcine common carotid) has recently been provided in [13]. A relative volume change of 5% was reported in the physiological pressure range of 50–200 mmHg that is lower in the case of the saphenous and femoral arteries (reported in [14]).

One of the basic assumptions in the modeling of hyperelastic soft biological tissues is to decompose the strain-energy function into volumetric and deviatoric parts. In the works of [15], it was shown that this decomposition is not physically realistic, especially for anisotropic behavior and relatively low strain levels. The investigation presented in [15] reveals that the stress distributions based on an arbitrary choice of Poisson‘s ratio close to 0.5 are very likely to be unrealistic.

A novel three-field formulation was developed in [16] for the numerical computation in the proximity of the simple inextensibility limit. In this novel formulation the volumetric–isochoric split of the deformation gradient, which is kinematically inadmissible, is avoided. The stress is decomposed into a uniaxial tension and a tensionless part also in the vicinity of the inextensibility limit.

Locking effects induced by materials at the nearly incompressible limit attracted a lot of research efforts. For this reason, the locking-free properties of high-order finite elements [17,18] at the limit close to incompressibility can be utilized for the modeling of soft tissues like arteries. The high convergence rate of the p-FEM [19,20] and the robustness with respect to large aspect ratios [21] in addition to the locking-free properties makes the p-FE method attractive for the modeling of soft tissues. A comparative study between the high-order and low-order finite elements focusing on accuracy and efficiency for hyperelastic materials can be found in [22].

High-order finite elements were utilized for the modeling of nearly incompressible hyperelastic anisotropic arteries in [23]. Here, the locking-free properties at the limit close to incompressibility, the high convergence rates, and the robustness with respect to large aspect ratios were demonstrated. By providing analytical solutions for several benchmarks for hyperelastic isotropic and anisotropic behavior, it has been shown that the p-FE implementation is much more efficient compared to the h-FEM in terms of number of degrees of freedom as well as CPU time.

A higher-order mixed finite element method has recently been provided for compressible transversely isotropic finite hyperelasticity in [24]. It has been considered that the anisotropy may develop from almost isotropic conditions into near-inextensibility at finite fiber tension. They also developed a generalized compressible transversely isotropic HGO model to avoid the volumetric–isochoric split.

Most of the research work on the mechanical modeling of arteries focuses on the passive response. Only very few works study the modeling of the active response of soft tissues.