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Optimal feedback control and controllability for hyperbolic evolution inclusions of Clarke's subdifferential type[☆]

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ABSTRACT

In this paper, by applying the weak closedness of multi-valued maps and the properties of Clarke's subdifferential, we consider new results of optimal feedback control and controllability for hyperbolic evolution inclusions of Clarke's subdifferential type. We omit the condition that the evolution operator and the values of multi-valued map are compact. Our main results can be applied to the problems of hyperbolic partial differential equations and hemivariational inequalities.

1. Introduction

Let X be a separable reflexive Banach space, Y be a reflexive Banach space and Y^* be the dual space of Y. In the sequel, we will study the problem with the following form:

$$\begin{cases} x'(t) \in A(t)x(t) + \gamma_2 \partial J(t, \gamma_1 x(t)), & t \in [0, T], \\ x(0) = x_0, \end{cases} \tag{1.1}$$

where $\{A(t)\}_{t\in[0,+\infty)}$ is a family of linear operators in X generating an evolution operator. $\gamma_1 : X \to Y$ is a linear compact operator and $\gamma_2 : Y^* \to X$ is a linear bounded operator. $J : [0, T] \times Y \to \mathbb{R}$ is a given function to be specified later. $\partial J(t, \cdot)$ denotes the Clarke's subdifferential of $J(t, \cdot)$.

The above problem leads to the hemivariational inequality (1.1) and is met, for example, in the nonmonotone nonconvex interior semipermeability problems. For the latter, Panagiotopoulos [1] considered a temperature control problem in which they regulated the temperature to deviate as little as possible from a given interval. We remark that the monotone semipermeability problems, leading to variational inequalities, have been studied by Duvaut and Lions in [2] under the assumption that $J(t, x, \cdot)$ is a proper, lower semicontinuous, convex function which means that $\partial J(t, x, \cdot)$ is a maximal monotone operator. For more details, one can see [3,4].

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Feedback control systems are ubiquitous around us, including trajectory planning of a robot manipulator, guidance of a tactical missile toward a moving target, regulation of room temperature, and control of string vibrations. Optimal feedback control of semilinear evolution equations and inclusions in Banach spaces has been studied [5–9].

As we know, the concept of controllability plays an important role in the analysis and design of control systems. Controllability of the deterministic and stochastic dynamical control systems in infinite-dimensional spaces is well developed by using different kinds of approaches, and the details can be found in various papers (see [10–12]). Many authors are devoted to the study of this field because it is useful to solve a lot of practical problems in applications.

However, the study for optimal feedback control and controllability for the systems described by hyperbolic evolution inclusions of Clarke's subdifferential type with noncompact evolution operator is still untreated topic in the literature and this fact is the motivation of the present work. In fact, the Clarke's subdifferential has important applications in mechanics and engineering, especially in nonsmooth analysis and optimization (see [3,13]). The evolution inclusions of Clarke's subdifferential type have been studied in many papers (see [6,14,15]).

In this paper, by applying the weak closedness of multi-valued maps, we shall assume that the evolution operator is strongly continuous instead of compact. Furthermore, by using the properties of Clarke's subdifferential, we study the results of existence, optimal feedback control and controllability for problem (1.1). From the above, our problems in this paper are valuable and it is worth to do further research on this subject.

The paper is organized as follows. In Section 2 we recall useful definitions and preliminaries. In Section 3 we obtain some new existence results. Theorems 3.7 and 3.8 are the main existence results of the paper. In Section 4, we give a result for optimal feedback control problem. In Section 5, we consider a controllability result. In the last section, we apply our problem to an example of a hyperbolic evolution inclusion with Clarke's subdifferential of type to illustrate our main results.

2. Preliminaries

Let X be a Banach space. The norm of X will be denoted by $\|\cdot\|_X$. For $T>0$, let $C([0,T];X)$ denote the Banach space of all continuous functions from $[0,T]$ into X with the norm $\|x\|_C=\sup_{t\in[0,T]}\|x(t)\|_X$ and $L^2([0,T];X)$ denote the Banach space of all square integrable functions from $[0,T]$ into X with the norm $\|x\|_{L^2}=\left(\int_0^T\|x(t)\|_X^2\,dt\right)^{\frac{1}{2}}$. We denote by "$\to$" the strong convergence and "$\rightharpoonup$" the weak convergence.

Let X and Y be two topological vector spaces. Denote by $P(Y)$ $[C(Y), C_v(Y)]$ the collections of all nonempty [respectively, nonempty closed, nonempty closed convex] subsets of Y. A multi-valued map $F:X\to C(Y)$ is said to be measurable, if $F^{-1}(D):=\{x\in[0,T]|F(x)\cap D\neq\emptyset\}\in\mathcal{Q}$ for every closed set $D\subset X$, where $\mathcal{Q}$ denotes the σ-field of Lebesgue measurable sets on $[0,T]$.

Definition 2.1. Let X and Y be two Banach spaces. A multi-valued map $F:X\to C(Y)$ is said to be

(i) upper semicontinuous (u.s.c. for short), if for every open subset $O\subset Y$ the set $F_+^{-1}(O)=\{x\in X: F(x)\subset O\}$ is open in X;
(ii) sequentially closed, if for any $(x_n,y_n)\in Gr(F):=\{(x,y)\in X\times Y: x\in X, y\in F(x)\}$ with $x_n\to\bar{x}$ in X, $y_n\to\bar{y}$ in Y we have $(\bar{x},\bar{y})\in Gr(F)$;
(iii) weakly sequentially closed, if for any $(x_n,y_n)\in Gr(F)$ with $x_n\rightharpoonup\bar{x}$ in X, $y_n\rightharpoonup\bar{y}$ in Y we have $(\bar{x},\bar{y})\in Gr(F)$;
(iv) compact, if it maps bounded set in X into relatively compact set in Y;
(v) weakly compact, if it maps bounded set in X into relatively compact set in Y_w.

For more details about multimaps, we refer to [16–19].

Now, let us proceed to the definition of the Clarke's subdifferential for a locally Lipschitz function $j:K\subset X\to\mathbb{R}$, where K is a nonempty subset of a Banach space X (one can see [3,13,20]). We denote by $j^0(x;y)$ the Clarke's generalized directional derivative of j at the point $x\in K$ in the direction $y\in X$, that is

$$j^0(x;y):=\limsup_{\lambda\to 0^+,\ \zeta\to x}\frac{j(\zeta+\lambda y)-j(\zeta)}{\lambda}.$$

Recall also that the Clarke's subdifferential or generalized gradient of j at $x\in K$, denoted by $\partial j(x)$, is a subset of X^* given by

$$\partial j(x):=\{x^*\in X^*: j^0(x;y)\geq\langle x^*,y\rangle,\ \forall y\in X\}.$$

Lemma 2.2 (*[13], Proposition 3.23*). *If $j:K\to\mathbb{R}$ is locally Lipschitz function, then*
(i) *the function $(x,y)\mapsto j^0(x;y)$ is u.s.c. from $K\times X$ into $\mathbb{R}$;*
(ii) *for every $x\in K$ the gradient $\partial j(x)$ is a nonempty, convex and weakly* compact subset of X^* which is bounded by the Lipschitz constant $L_x>0$ of j near x;*
(iii) *the graph of ∂j is closed in $X\times X^*_{w^*}$;*
(iv) *the multi-valued map ∂j is u.s.c. from K into $X^*_{w^*}$.*

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