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Generalized conjugate direction method for solving a class of generalized coupled Sylvester-conjugate transpose matrix equations over generalized Hamiltonian matrices[☆]

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ABSTRACT

In this paper, a generalized conjugate direction (GCD) method for finding the generalized Hamiltonian solutions of a class of generalized coupled Sylvester-conjugate transpose matrix equations is proposed. Furthermore, it is proved that the algorithm can compute the least Frobenius norm generalized Hamiltonian solution group of the problem by choosing a special initial matrix group within a finite number of iterations in the absence of round-off errors. Numerical examples are also presented to illustrate the efficiency of the algorithm.

1. Introduction

The special solution of matrix equations has raised much interest among researchers due to the wide applications such as robust control, neural network, singular system control, model reduction and image processing [1–4]. For example, we need to solve the (coupled) Sylvester matrix equations over symmetric matrices when finite element techniques are designed to model the vibrating structures such as highways, bridges, buildings and automobiles, for details see [5–9]. In [10], the generalized eigenvalue problems lead to attention of the solution pair (X, Y) of the generalized coupled Sylvester matrix equations

$$\begin{cases} AX - YB = C, \\ DX - YE = F. \end{cases}$$

So far, many researches are devoted to the special solutions of several matrix equations. Li, H. et al. (see [11]) considered the least squares solution of the generalized Sylvester matrix equations

$$AXB + CYD = E. \tag{1.1}$$

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Furthermore, M. Hajarian (see [12]) proposed the CGLS algorithm for least squares solutions of the generalized Sylvester-transpose matrix equations

$$\sum_{i=1}^{s} A_i X B_i + \sum_{j=1}^{l} C_j X^T D_j = E. \tag{1.2}$$

Least Squares Solution of Linear Operator Equations was also investigated (see [13]).

Ding and Chen (see [14,15]) considered gradient based iterative algorithms to solve the generalized couple Sylvester matrix equations. In [16–20], the matrix form of CGS, Bi-CGSTAB, BICOR and QMRCGSTAB was presented. Dehghan and Hajarian [4] considered the generalized coupled Sylvester matrix equations

$$\sum_{i=1}^{l} A_i X B_i + \sum_{i=1}^{l} C_i Y D_i = M. \tag{1.3}$$

and

$$\sum_{i=1}^{l} E_i X F_i + \sum_{i=1}^{l} G_i Y H_i = N. \tag{1.4}$$

In [21], the author constructed a conjugate direction method for the generalized nonhomogeneous Yakubovich-transpose matrix equation

$$AXB + CX^T D + EYF = R, \tag{1.5}$$

and obtained the (least Frobenius norm) solution pair (X, Y).

Recently, some extended conjugate gradient algorithms are investigated for solving various matrix equations over symmetric (antisymmetric), reflexive (irreflexive) and generalized bisymmetric matrices [22–26]. Masoud Hajarian in [27] focused on the symmetric solution group of the general coupled matrix equations

$$\sum_{j=1}^{m} A_{ij} X_j B_{ij} = C_i, \quad i = 1, 2, \ldots, n, \tag{1.6}$$

where $A_{ij} \in R^{p_i \times n_j}$, $B_{ij} \in R^{n_j \times q_i}$, $C_i \in R^{p_i \times q_i}$ and $X_j \in R^{n_j \times n_j}$ for $i = 1, 2, \ldots, n$ and $j = 1, 2, \ldots, m$.

Motivated by [27], this paper first investigates the generalized coupled Sylvester-conjugate transpose matrix equations as follows

$$\sum_{j=1}^{l} \left(A_{ij} X_j B_{ij} + C_{ij} X_j^H D_{ij} \right) = E_i, \ i = 1, 2, \ldots, s, \tag{1.7}$$

where $A_{ij}, C_{ij} \in C^{m \times n}$, $B_{ij}, D_{ij} \in C^{n \times r}$ and $E_i \in C^{m \times r}$, $i = 1, 2, \ldots, s$, $j = 1, 2, \ldots, l$, are given matrices and $X_j \in C^{n \times n}$, $j = 1, 2, \ldots, l$, are unknown matrices to be determined. Eq. (1.7) has wide applications in many fields, such as in vibration and structural analysis, robotics control and spacecraft control. Especially, it is very important for obtaining the least Frobenius norm generalized Hamiltonian solution group of the problem (1.7). Hence, we present a generalized conjugate direction method (CD algorithm) for solving Eq. (1.7) over generalized Hamiltonian matrix.

The rest of this paper is organized as follows: In Section 2, we present the generalized conjugate direction method (GCD method) to solve the generalized coupled Sylvester-conjugate transpose matrix equations (1.7) as the system is consistent. The convergence properties of the GCD method are reported later; In Section 3, we give a special choice of initial matrix group and show that the least Frobenius norm generalized Hamiltonian solutions can be obtained consequently within finite iterative steps in the absence of roundoff error; Some numerical results are reported in Section 4; The conclusions are given in Section 5 at last.

In our notation, let $R^{m \times n}$ and $C^{m \times n}$ be the sets of all real and complex $m \times n$ matrices, respectively. Let $A \in C^{m \times n}$, we write $Re(A)$, $Im(A)$, $\overline{A}$, A^T, A^H, $\|A\|_F$, A^{-1}, and $\mathcal{R}(A)$ to denote the real part, imaginary part, conjugation, transpose, conjugate transpose, Frobenius norm, inverse, and the column spaces of matrix A, respectively. For any matrix $A = (a_{ij})$, $B = (b_{ij})$, matrix $A \bigotimes B$ denotes the Kronecker product defined as $A \bigotimes B = (a_{ij} B)$. For the matrix $X = (x_1, x_2, \ldots, x_n) \in \mathbb{C}^{n \times n}$, $\mathrm{vec}(X)$ denotes the vec operator defined as $\mathrm{vec}(X) = (x_1^T, x_2^T, \ldots, x_n^T)^T \in C^{mn}$. Let $ASOR^{m \times m}$ stand for the sets of all $m \times m$ antisymmetric orthogonal matrices, i.e.,

$$ASOR^{m \times m} = \{J \,|\, J^T J = JJ^T = I_m, J = -J^T, J \in R^{m \times m}\}.$$

In the space $C^{m \times n}$, the inner product can be defined as

$$\langle A, B \rangle = Re[tr(A^H B)]. \tag{1.8}$$