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Some periodic type solutions for stochastic reaction–diffusion equation with cubic nonlinearities ☆

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ABSTRACT

In this paper, we discuss the bounded solutions, stationary solutions, periodic solutions, almost periodic solutions, almost automorphic solutions for stochastic reaction–diffusion equation with cubic nonlinearities. The main difficulty is the cubic nonlinearities, we overcome this difficulty by the semigroup approach, the energy estimate method and refined inequality technique.

1. Introduction

Reaction–diffusion equations

$$u_t - \triangle u + u^3 - bu = 0$$

are important mathematical models naturally applied in fields such as chemistry, biology, geology, physics and ecology. In the deterministic context, they can model various phenomena such as the models of predators and preys (Lotka Volterra models) [1,2] or describe the free propagation of particles/molecules from a transmitter to a receiver in molecular communication [3,4], or model the birth–death processes at population levels [5].

The theory of stochastic partial differential equations has attracted much attention due to its wide applications in the fields such as financial market, insurance, biology, medical science, population dynamic, control, and so on.

Stochastic reaction diffusion equations have vast applications in biology, ecology, neuroscience, nanobioscience, etc. In the stochastic versions, they can e.g. model metabolic processes [6], birth–death processes and random movements at the organism levels [7,8], etc. The stochastic diffusion systems with polynomial growth reaction terms, that we consider here, can be e.g. Lotka Volterra systems driven by multiplicative white noise. These problems have been widely studied, in the settings of predators and preys, see e.g. [9,10] or stochastic systems of neurons.

Reaction–diffusion systems and stochastic perturbations of them play an important role in applications in chemistry, biology and physics [11].

In this paper, we discuss the following system

$$\begin{cases} du + (-\triangle u + u^3 - bu)dt = g(t,u)dB & \text{in } Q = D \times \mathbb{R}, \\ u(x,t) = 0 & \text{in } \partial D \times \mathbb{R}, \end{cases} \tag{1.1}$$

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where D is a bounded region in $\mathbb{R}^n (n \geq 1)$ having smooth boundary ∂D.

The study of the qualitative behavior of evolution equations is one of the most important problems in the field of differential equations, as the vast literature on the subject shows.

The qualitative behavior of the stochastic differential equations is an active research field on which there is a great deal of literature. Taking into account the generalized and refined results, it is worthy quoting the papers

- Stochastic periodic solutions: [12,13];
- Stochastic almost periodic solutions: [12,14–16];
- Stochastic almost automorphic solutions: [17].

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We are also referred to [13] and the references therein for recent related work on stochastic systems in finite dimensional space.

In recent years, there are some results on the qualitative behavior for stochastic evolution equations in infinite dimensional space. To this purpose we recall the recent results:

- Stochastic periodic solutions: [18–20];
- Stochastic almost periodic solutions: [12,18,21–26];
- Stochastic almost automorphic solutions: [27–30].
- Stochastic pseudo almost automorphic solutions: [31–33].

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The above papers all consider the stochastic evolution equations, the nonlinear terms are assumed to be Lipschitz continuous and in particular to have linear growth. However, there are very few results for the qualitative theory and qualitative behavior for stochastic reaction–diffusion equation.

Motivated by previous research and from both physical and mathematical standpoints, the following mathematical questions arise naturally which are important from the point of view of dynamical systems:

- *Does (1.1) have bounded solutions, stationary solutions, periodic solutions, almost periodic solutions and almost automorphic solutions?*

In this paper we will answer the above problem. There are many results on qualitative behavior of solutions in the stochastic case, but most treat only stochastic evolution equations, and those generalizations which do treat the infinite-dimensional case usually assume a global Lipschitz property for the nonlinear terms of the non-random equation. In our case the nonlinear term u^3 is only locally Lipschitz, it is easy to see (1.1) does satisfy the assumptions in the above references, so that these stochastic results cannot be applied directly to our problem.

Our main novelty of this paper is

★ Overcome the difficulty of the cubic nonlinear term u^3. The usual Banach fixed point theorem cannot work in our problem, our approach is completely different from theirs as above, motivated by [34], we overcome this difficulty by the semigroup approach, the energy estimate method and refined inequality technique. The crucial tools are Propositions 3.1 and 3.3 which play a vital role in this article. To the best of our knowledge, it is the first contribution to the literature on this problem.

We introduce the following mathematical setting:

◇ Let $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geq 0}, P)$ be a complete filtered probability space. Let Y be a Banach space, and let $C([0,T];Y)$ be the Banach space of all Y-valued strongly continuous functions defined on $[0,T]$. We denote by $L^p_{\mathcal{F}}(0,T;Y)(1 \leq p < +\infty)$ the Banach space consisting of all Y-valued $\{\mathcal{F}_t\}_{t\geq 0}$-adapted processes $X(\cdot)$ such that $E(\|X(\cdot)\|^p_{L^p(0,T;Y)}) < \infty$; by $L^2_{\mathcal{F}}(\Omega; C([0,T];Y))$ the Banach space consisting of all Y-valued $\{\mathcal{F}_t\}_{t\geq 0}$-adapted continuous processes $X(\cdot)$ such that $E(\|X(\cdot)\|^2_{C([0,T];Y)}) < \infty$. All the above spaces are endowed with the canonical norm.

◇ $(\cdot,\cdot)$ stands for the inner product in $L^2(D)$, $\|\cdot\|$ stands for the norm in $L^2(D)$. $H \triangleq L^2(D)$.

We assume that U is a separable Hilbert space. The set of Hilbert–Schmidt operators from Hilbert space U into H will be denoted by $L_2(U,H)$ acting between U and H equipped with the Hilbert–Schmidt norm $\|\cdot\|_2$. Let $Q \in L_2(U,H)$ be a symmetric nonnegative operator with finite trace, $U_0 = Q^{\frac{1}{2}}U$ and $L_2^0 \triangleq L_2(U_0,H)$, which is a separable Hilbert space with respect to the norm

$$\|\Phi\|^2_{L_2^0} = \|\Phi Q^{\frac{1}{2}}\|_2^2 = Trace(\Phi Q \Phi^*).$$

$L_2^1 \triangleq L_2(U_0, H_0^1(D))$.

◇ Let $\{B_i(t)\}_{t\geq 0}(i = 1,2)$ be two independent Q-Wiener processes with values in U with $Q^{\frac{1}{2}} \in L_2^1$. Define the U-valued process $\{B(t)\}_{t\geq 0}$ by

$$B(t) = \begin{cases} B_1(t), & t \geq 0, \\ B_2(t) & t \leq 0, \end{cases}$$

we denote $\mathcal{F}_t = \mathcal{B}(B_u - B_v : -\infty \leq u \leq v \leq t)$.

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