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Simulations of a fractional rate type nanofluid flow with non-integer Caputo time derivatives

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ABSTRACT

Unsteady developed flow of a rate type anomalous nanofluid with non integer Caputo fractional derivatives is studied numerically in this article. Mixed convection and diffusion are taken into account while analyzing transport phenomena in the flow field. Thermophoresis and pedesis effects are also incorporated. Fluid is confined between nonisothermal parallel plates and flows by the moving lower plate. Variable concentration is assumed at both the plates. In literature no such result exists with non integer Caputo fractional derivatives. Boundary layer flow is modeled with the help of fractional calculus approach. Governing flow partial differential equations with appropriate conditions are solved by finite difference-finite element scheme. Given scheme is flexible for the solution of non-linear flow problems. Local Nusselt and Sherwood numbers are computed for the fractional model. Flow behavior is presented for various values of involved parameters. Influence of different dimensionless quantities on the Nusselt and Sherwood numbers is discussed by tabular results. The acquired results revealed that fractional exponents α, β have opposite effects on the velocity profiles. It is also noted that thermophoresis and pedesis parameters have similar effects on heat flux while opposite effects are observed for mass flux at both the plates. Various stretching flows particularly in paper, polymeric and food production processes can be modeled in a similar manner.

1. Introduction

Heat and mass transfer are the kinetic processes that can be studied jointly or separately. Usually these processes are considered as analogue of each other but in case of convection and diffusion there is no mass transfer similarity to heat radiation. Apart from gas fuel jets, condensed fuels there are some cases where heat and mass transfer must be considered jointly like ablation and evaporative cooling. So it is efficient to consider both the processes simultaneously because they have similar mathematical formulation. Transport processes have a potential role on thermal systems management, thermal control for engineering, technological and industrial applications. Particularly, investigation of transport phenomena in non-Newtonian fluids is an active area of research due to their occurrence in modern science and technology including density machines with high power, heat exchangers, granular insulation, fiber technology, thermal energy devices, crystal growth, nuclear based repositories, geothermal energy extraction, fermentation processes, petroleum reservoirs, production of crude oil etc. Transport phenomena is discussed extensively by various researchers. For instance, Sheikholeslami [1] discussed heat transfer in nanofluid through open porous cavity. Sheikholeslami [2] analyzed nanofluid flow with free convection and Darcy effects. Investigation of heat transfer within Carreau fluid flow with radiation is studied by Khan et al. [3]. Hashim

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and Khan [4] discussed heat flux for Carreau fluid model. Sheikholeslami [5] presented numerical study of nanofluid flow through porous medium with magnetic field effects. Mass transfer adaptive mesh method for two phase fluid flows is studied by Sulman [6]. Hayat et al. [7] investigated mass transfer in MHD flow of second grade fluid over a moving cylinder. Non-Newtonian fluids are mainly classified into integral, rate and differential types. In rate type fluids stress is an implicit function of velocity gradient and its higher derivatives of time. In this article, we will consider the Oldroyd-B fluid which is a subclass of rate type fluid models. Rate type fluid models have extensive applications in fiber technology and polymer processing. Oldroyd-B model can define the fluid in terms of its relaxation and retardation times. Many fluid flows can be modeled with this model such as slurry flows, dilute polymer solutions and industrial oils. Shehzad et al. [8] analyzed three dimensional flow of Oldroyd-B nanofluid. Generalized Oldroyd-B helical flow through porous medium is discussed by Li et al. [9]. Flow of fractional Oldroyd-B fluid is examined by Riaz et al. [10]. Hayat et al. [11] investigated MHD flow of Oldroyd-B nanofluid with heat generation. In past few years transport phenomena is discussed by various researchers [12–19].

In past few decades fractional calculus gained high importance in various fields of science and engineering. Valuable impact of this branch of calculus is observed in viscoelasticity, control theory, mechanics, electrochemistry and mechatronics [20]. For technology professionals, the fractional derivatives are more significant than traditional derivatives because of their importance in understanding the working of artificial and natural systems. Fractional calculus is currently used as an applied technique in various branches of science. Fractional models are better than integer order models because they describe the hereditary and memory properties of diverse substances [21]. Anomalous non-Newtonian fluid models can more accurately predict the experimental data than usual rheological fluid models [22]. Fractional time derivatives are used to model the anomalous behavior of fluids [23]. In these fluids flows space fractional derivatives describe extended motions through fractures while time fractional derivatives model particles that persist motionless for long period of time [22]. Unidirectional flow of fractional Jeffrey fluid with thermodynamic constraints is studied by Emilia Bazhlekova and Ivan Bazhlekova [24]. MHD flow of a fractional Oldroyd-B fluid is presented by Liu et al. [25]. Churbanov and Vabishchevich [26] investigated a turbulent flow for fractional model in rectangular ducts. Finite element approach for the solution of fractional Oldroyd-B fluid is discussed by Rasheed et al. [27].

Now a days nanotechnology is significantly important due to their wide range of applications in production of lightweight materials, treatment of chronic bacterial infections, industrial coatings to protect wood, stimulate immune responses, making clothing odor-resistant, bone growth around dental or joint implants, breakdown of volatile organic pollutants in air etc. Nanoparticles have at least one dimension that measures 1–100 nm and act as a bridge between bulk and atomic structures. Suspension of these particles are possible because interaction of nanoparticles surface with the base fluid is strong enough to overcome the difference in density. Properties of base fluid changes with the nanoparticles. For example nanoparticles suspension increases thermal conductivity of the base fluid. Flow of nanofluids is an intense area of scientific research. Wang et al. [28] analyzed heat transfer in nanofluids flows. Mechanism for the enhancement of heat transfer by nanoparticles suspension is discussed by Vadasz et al. [29]. Paul et al. [30] examined concentration of water based disperse nanofluids. Mehmood et al. [31] investigated energy transfer in a nanofluid flow over a moving plate. Nanofluid flow through moving channel is presented by Khan et al. [32]. Recently, nanofluids flows are discussed in various flow regimes [33–39]. But the fractional flow of nanofluids is not given appropriate attention due to the presence of highly non-linear terms. In literature fractional flows are mostly solved with the help of Fourier sine, Fourier cosine and Laplace transforms. It is difficult to obtain the exact solution of coupled non-linear equations by the existing methods. Mostly governing equations of non-Newtonian models are non-linear in nature. In general exact solutions of these models are not obtained by present techniques. But numerical schemes are extensively used in order to solve rheological non-linear models [40–50,52]. Here we used finite difference-finite element scheme to solve the present fractional flow configuration of anomalous Oldroyd-B fluid. Graphical results demonstrate the convergence of the solution for various values of involved parameters.

The main idea of this study is to elaborate the importance of fractional derivatives in Oldroyd-B nanofluid flow with mixed convection and diffusion. The current analysis is performed for variable temperature and concentration at the boundaries. The flow is considered between parallel plates of infinite length. The fluid flows when the lower plate start moving with linear acceleration. Temperature is assumed to vary non-linearly at the lower boundary while linearly at the upper boundary. No slip condition is imposed at the boundaries. Convergent numerical solution of the governing mathematical model is presented via finite element method and finite difference approximation is used for fractional time derivatives. Effects of various fractional flow parameters over the velocity, temperature and concentration profiles are analyzed graphically. Here our main goal is to indicate the behavior of fractional Oldroyd-B nanofluid using stabilized finite element-finite difference scheme. Flow is modeled in the presence of fractional exponents, nanoparticles, thermophoresis and pedesis effects. In Section 2, mathematical formulation of the flow problem is presented. Approximation of the velocity field by means of finite difference-finite element scheme is given in Section 3. Graphical results are given in Section 5. At the end conclusions are given in Section 6.

2. Mathematical modeling

Consider unsteady flow of a fractional rate type nano-fluid between infinite parallel plates with mixed convection and diffusion. Space between the plates is filled with a rate type fluid with colloidal suspension of tiny particles in a base fluid. Plates are separated by a distance $2L > 0$. At $t = 0$ the fluid and plates are at rest. Also initially both plates and the fluid are at constant temperature θ_0 and concentration ϕ_0. When $t > 0$ fluid starts moving by the motion lower plate. Acceleration of the