Contents lists available at ScienceDirect

Computers and Mathematics with Applications

journal homepage: www.elsevier.com/locate/camwa

On the formulation and application of design rules

Barna Szabó [a,*], Ricardo Actis [a], David Rusk [b]

[a] Engineering Software Research and Development, Inc., USA
[b] NAVAIRSYSCOM, USA

ARTICLE INFO

Article history:
Available online xxxx

This paper is dedicated to Professor Ivo Babuška on the occasion of his 90th birthday

Keywords:
Certification
Numerical simulation
Simulation governance
Metal fatigue
Statistical models
Uncertainty quantification

ABSTRACT

Design rules are stated in the form of a predictor of failure initiation and its allowable value in the context of design of mechanical and structural components subjected to cyclic loading. Calibration and validation procedures are described and illustrated for the predictors with the aid of published experimental data. It is noted that mathematical models cannot be validated with respect to the allowable value of predictors.

1. Introduction

The formulation and application of design rules are central to engineering practice. This paper is concerned with design rules used in mechanical, structural and aerospace engineering.

Engineers responsible for design and certification are required to apply established rules typically stated in the form:

$$F_{max} \le F_{all} \tag{1}$$

where F_{max} is the maximum value of a predictor of failure F such as stress, strain, stress intensity factor, buckling load, etc. and F_{all} is the allowable value of F, established by authorities charged with the responsibility to formulate design rules.

The development of design rules is a continuing activity driven by evolving requirements, material systems and manufacturing techniques. For example, the American Society of Mechanical engineers (ASME), responding to several major accidents, charged a committee with the task to establish rules for the design of boilers in 1911. This committee, now called the ASME Boiler Code Committee, published the first edition of the ASME Boiler Code in 1915, a 144-page document, that evolved into the Boiler and Pressure Vessel Code (BPVC) which currently consists of 16,000 pages in 28 volumes and has been adopted by over 100 countries [1].

In the aerospace sector much of the shared information relating to design of metallic structural components is maintained in the Metallic Materials Properties Development and Standardization (MMPDS) Handbook [2] previously known as MIL-HDBK-5. MMPDS, distributed by Battelle Memorial Institute, is a source of statistically based design values for commonly used metallic materials and joints. In addition, many design rules and procedures are developed by the manufacturers. Sharing those design rules and the supporting experimental data is largely restricted by proprietary considerations.

* Corresponding author.
E-mail addresses: barna.szabo@esrd.com (B. Szabó), ricardo.actis@esrd.com (R. Actis), david.rusk@navy.mil (D. Rusk).

http://dx.doi.org/10.1016/j.camwa.2017.06.040
0898-1221/

We examine the formulation of design rules in the context of design of metallic structural and machine elements subjected to cyclic loading. Referring to Eq. (1), the goal is to establish allowable levels of a predictor F, that can be correlated with the formation of cracks caused by metal fatigue for specific alloys. Here we consider high cycle fatigue, that is, fatigue failure occurs above 10^4 load cycles.

The physical processes that lead to the formation of cracks are highly complex, irreversible processes on the microscopic scale. These processes violate the assumptions of small strain continuum mechanics. Nevertheless, the predictors used for correlating the formation of cracks with the number of load cycles in high cycle fatigue are usually determined from solutions of problems of linear elasticity. This apparent contradiction is resolved through (a) the understanding that stresses (or strains) computed from the solutions of problems based on the linear theory of elasticity represent average stresses or strains over a representative volume element rather than pointwise stresses and (b) the assumption that those average stresses can be correlated with crack initiation events through suitably chosen predictors calibrated to experimental observations.

The mathematical model described herein has three constituent parts: (a) A mathematical problem based on the linear theory of elasticity, (b) a family of predictors formulated for the correlation of failure initiation events with the stress field and (c) a statistical model that accounts for the statistical dispersion of the experimental data. Predictors and statistical models are phenomenological sub-models that are calibrated on the basis of experimental data. Many plausible predictors and statistical models can be formulated. The ranking of competing mathematical models on the basis of their predictive performance is discussed in [3].

In this paper we focus on the formulation and calibration of a predictor used in conjunction with two statistical models that belong in the class of random fatigue limit models [4].

This paper is organized as follows: A family of predictors of fatigue damage in the high cycle range is defined. The statistical models and the predictors are calibrated against published experimental data and ranking of mathematical models is discussed. It is observed that mathematical models can be validated with respect to events that have high probability of occurrence but cannot be validated with respect to events that have a very low probability of occurrence. Therefore mathematical models cannot be validated with respect to the allowable values of the predictors. The application of design rules is discussed noting that assurance of numerical accuracy is an essential technical requirement. Finally, it is noted that it is possible to improve the predictive performance of mathematical models over time through proper management of simulation resources and experimental data.

2. Predictors of fatigue life

The formulation of predictors of fatigue failure is based on intuition and experience. General rules are that a predictor must be independent of the choice of the coordinate system and small changes in the characterizing data must not produce large changes in the predicted quantity.

The predictors used in classical fatigue life estimation depend on the maximum normal stress $\sigma_{\max}$ and the stress amplitude σ_a defined by

$$\sigma_a = \frac{\sigma_{\max} - \sigma_{\min}}{2} = \sigma_{\max} \frac{1-R}{2} \tag{2}$$

where R, called the cycle ratio, is the ratio of the minimum principal stress $\sigma_{\min}$ to the maximum principal stress at the location of the maximum principal stress. When the linear theory of elasticity is applicable then it is also the ratio of the minimum applied load to the maximum applied load. A family of predictors widely used in engineering practice for uniaxial stress is:

$$F = \sigma_{\text{eq}} = \sigma_{\max}^{1-q}\,\sigma_a^{q}, \quad 0 < q < 1 \tag{3}$$

where σ_{eq} is called equivalent stress. Using Eq. (2) we have:

$$F = \sigma_{\text{eq}} = \sigma_{\max}\left(\frac{1-R}{2}\right)^{q}, \quad 0 < q < 1. \tag{4}$$

We denote the number of cycles at failure by N. Plotting σ_{eq} against $\log_{10} N$ for fatigue tests conducted at various R values results in a cluster of points that can be approximated by a single curve, called the S–N curve. Methods for the determination of S–N curves for various statistical models and q values are described in [5]. The special case of $q = 1/2$ is one of the predictors proposed in [6]. In this paper we also use $q = 1/2$.

Predictors are formulated with the objective to generalize results of fatigue tests performed on coupons subjected to constant or smoothly varying uniaxial or, less frequently, biaxial stress conditions to arbitrary triaxial stress conditions.

Specifically, we will investigate the performance of a family of predictors defined as follows:

$$F = G_\alpha = \frac{1}{V_c}\int_{\Omega_C} (\alpha I_1 + (1-\alpha)\bar{\sigma})\, dV \left(\frac{1-R}{2}\right)^{1/2}, \quad 0 \le \alpha \le 1 \tag{5}$$

where I_1 is the first stress invariant, $\bar{\sigma}$ is the von Mises stress, α is a material-dependent parameter determined by calibration and V_c is the volume of the domain of integration defined by

$$\Omega_C = \{\mathbf{x} \,|\, \sigma_1 > \beta\sigma_{\max}\} \tag{6}$$