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A new rapid auto-adapting diffusion function for adaptive anisotropic image de-noising and sharply conserved edges

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ABSTRACT

Anisotropic diffusion, based on partial differential equation (PDE), is a recent adequate solution for the problem of image filtering. The first works in this context are those of Perona and Malik. Recently, several studies have shown the drawbacks of this approach such as the "staircase" effect and flow edges caused by the slow convergence of the diffusion function. In this work, we suggest a new diffusion function, which converges faster than that of Perona and Malik. The suggested function decreases rapidly to disappear once borders or details are detected. This rapidity to expand and converge to zero allows us to implement a real time processing device. Moreover, the suggested model is able to remove the "staircase" effect, preserve sharp transition and discontinuities and remove noise efficiently. The diffusion barrier is chosen to get rid of the noise and enhance the edges. Extensive experiments on several standard test images are conducted to compare our algorithm with other well-known algorithms. Experimental results are very interesting and show the efficiency of the suggested method based on a comparison study.

1. Introduction

Since the image is often infected by noise, the improvement of the image quality has now become a necessity in many application domains such as the military and medical. The main goal of de-noising is to remove the noise while retaining as much signal features as possible. A vast literature has emerged recently on image restoration using linear or nonlinear techniques. For instance, anisotropic diffusion methods have become a very useful tool in image smoothing, edge detection, image segmentation, and image enhancement [1] and [2]. Anisotropic diffusion filtering can successfully smooth noise while respecting the region boundaries and small structures within the image. It depends on the norm of the gradient which occurs in the so-called diffusivity function in the heat equation in order to control the diffusion. This diffusivity function $C\,\|\nabla U\|$ is between [0 1]. If the diffusivity coefficient called C is equal to zero, the distribution will be stopped. Conversely, when C is equal to "1", it is isotropically scattered by the heat equation. The most well-known diffusion function was suggested by Perona and Malik [3]. Despite convincing experimental results, several authors have raised the drawbacks of this approach. In case of a high-level noise, this approach intensifies it. This is due to the non-convergence speed function diffusivity, also it tends to cause the so called "staircase" effect [4] and [5]. To overcome this problem, a great amount of studies have been conducted which led to positive results. Catté, Lions, Morel and Coll introduce a convolution of the image with a Gaussian kernel of standard deviation σ before each iteration [6] and [7], Alvarez, Lion and Morel suggested to solve the following

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PDE [8].

$$\begin{cases} \dfrac{\partial u}{\partial t}(x,y,t) = g(\|\nabla u_\sigma\|)\,\|\nabla u\|\,div\left(\dfrac{\nabla u}{\|\nabla u\|}\right) \\ u(x,y,0) = u_0(x,y) \end{cases} \quad (1)$$

where μ_σ is the smoothed image with a Gaussian kernel of standard deviation σ. Kornprobst suggested a very general reaction–diffusion model that combines a shock filter to a selective smoothing. The equation of the model contains a diffusion term associated with the measurement of the average curvature, a feedback term based on the shock filters theory developed by Osher and Rodin [9] and a term attached to the data to obtain a solution close to the original image [8]. In other work, You et al. in [10] suggest to solve the following diffusion function which eliminates the "staircase" effect [11].

$$\begin{cases} g(x) = \dfrac{1}{K} + p(K+\varepsilon)^{p-1} \quad if\ x < K \\ g(x) = \dfrac{1}{x} + p(x+\varepsilon)^{p-1} \quad if\ x > K. \end{cases} \quad (2)$$

In related work [12], authors proposed to remove noise from polluted images such as classical total variation scheme (TV model) which gives a better effect with tiny staircases to a certain degree through the using of gradients alone. Recently, an anisotropic diffusion model with a semi-adaptive threshold in diffusion coefficient function (SAT) is applied to get a restored image [13]. In this work, we suggest a new diffusion function $C\,\|\nabla U\|$ with better mathematical characteristics which converges faster than the one of the Perona and Malik. This new model can alleviate the "staircase "effect, preserve sharp discontinuities and remove noise simultaneously. The diffusion barrier is chosen so that noise is removed and edges are enhanced. The significant advantage is the possibility to implement real time processing based on this algorithm thanks to its rapidity of convergence. The experimental results are promising and encouraging. A comparative study is also provided to prove the efficiency of the suggested method.

The paper is organized as follows. In the next section, we first introduce the anisotropic diffusion. An efficient model proposed by Perona and Malik is then reviewed. In Section 3, we review the limitations of Perona and Malik scheme and we cite the recent work to overcome this problem, then we present the new model. Section 4 describes the results and Section 5 describes the results evaluation. Section 6 is devoted to the conclusion of this paper.

2. Anisotropic diffusion and Perona and Malik formulation

Anisotropic Diffusion is an example of scale-space image processing [3]. It has transformed the application of the heat diffusion equation to digital images. Considering an image U, the heat flux ϕ within the image is given by Fick's equation [14].

$$\phi = -D \cdot \nabla U \quad (3)$$

where: D is diffusion tensor in general it is a definite positive, symmetric matrix expresses the relationship between the gradient ∇U and the actual flux ϕ.

Since the diffusion process does not alter the overall energy in the image, its local variation is driven by $\delta_t = -div\phi$, where div stands for the divergence operator [14] and [15]. Hence, we have:

$$\delta_t U = div(D \cdot \nabla U). \quad (4)$$

In relation to the image, Eq. (2) expresses the energy (heat) variation at each pixel position. The diffusion algorithm smoothes the image based on partial differential equation 'PDE' [16].

Perona and Malik applied an anisotropic process that reduces the diffusivity and consequently the smoothing strength proportionally to the probability of edge presence.

The method is to evolve an image under the following PDE.

$$\frac{\partial U}{\partial t} = div\,[c(x,y,t)\nabla U(x,y,t)] \quad (5)$$

$$\text{with} U(x,y,0) = U_0(x,y). \quad (6)$$

The conductivity $c(x,y)$ is developed to promote smoothing intra regional and penalize inter regional diffusion through the use of a decreasing function dependent on the magnitude of the vector gradient.

$$c(x,y,t) = g\left(|\nabla U|\right). \quad (7)$$

The functions suggested initially by Perona and Malik [17]:

$$g_1\left(|\nabla U|\right) = \exp\left[-\left(\frac{|\nabla U|}{K}\right)^2\right] \quad (8)$$

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