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Distributionally robust fixed interval scheduling on parallel identical machines under uncertain finishing times

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A R T I C L E I N F O

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A B S T R A C T

We deal with fixed interval scheduling (FIS) problems on parallel identical machines where the job starting times are given but the finishing times are subject to uncertainty. In the operational problem, we construct a schedule with the highest worst-case probability that it remains feasible, whereas in the tactical problem we are looking for the minimum number of machines to process all jobs given a minimum level for the worst-case probability that the schedule is feasible. Our ambiguity set contains joint delay distributions with a given copula dependence, where a proportion of marginal distributions is stressed and the rest are left unchanged. We derive a trackable reformulation and propose an efficient decomposition algorithm for the operational problem. The algorithm for the tactical FIS is based on solving a sequence of the operational problems. The algorithms are compared on simulated FIS instances in the numerical part.

1. Introduction

In this paper, we deal with distributionally robust fixed interval scheduling problems that can be characterized as follows. The number of jobs together with their start and prescribed end times, when each job is expected to be finished, are known in advance. However, the actual end times are subject to uncertainty which is taken into account using random delays. Preemption of jobs is not allowed; once a job is assigned to a machine, it must be finished by that machine and cannot be transferred to another machine. Moreover, no job is allowed to be preempted and resumed later on the same machine. Each job must be processed by one machine, and each machine can process at most one job at a time. All machines are identical, i.e., the processing times including the distribution of random delays do not depend on the processing machine. The probabilistic distribution is not known precisely, and it is assumed to belong to an ambiguity set which contains possible distributions of random delays. The problems lie in finding "here-and-now" decisions on assigning the jobs to machines at the beginning of the planning horizon. If the schedule becomes infeasible during its processing, a re-optimization is necessary. Note that an earlier end, which is also possible in practice, does not influence the schedule feasibility because the starting times of the subsequent jobs are given and cannot be moved. Thus, only positive delays can cause problems with the schedule feasibility.

We will focus on two variants of the robust FIS problem. In the operational problem, we would like to construct a schedule with the highest worst-case probability that it remains feasible. In the tactical problem, we are looking for the minimum number of machines to process all jobs given a minimum level for the worst-case probability that the schedule is feasible. The worst-case probability is related to the most pessimistic choice of the probability distribution from a particular ambiguity set. The stochastic integer programming formulations of these problems are provided in Section 2.

We will discuss a possible application to the gate assignment problem where a set of flights is assigned to available gates. Based on historical observations, we can estimate the parameters of the delay distribution. However, there can be a proportion of gate assignments, possibly time-period dependent, for which a worse delay appears. This can be incorporated into the FIS problem by a suitable choice of the ambiguity set of the delay distributions. The operational problem then leads to a flight assignment to available gates with the highest attainable worst-case probability that the schedule remains feasible, whereas the tactical problem aims at

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finding the optimum number of gates to fulfill a prescribed worst-case probability for the schedule feasibility.

We briefly review the recent literature on FIS. Older results on deterministic FIS problems were summarized by two survey papers Kolen et al. (2007), Kovalyov et al. (2007). Eliiyi (2013) discussed complexity and algorithms for the operational and tactical problems and identified several polynomially solvable special cases. Angelelli et al. (2014) compared exact and heuristic approaches to solving a FIS problem with additional resource constraints, which was earlier shown to be NP-hard by Angelelli and Filippi (2011). Ng et al. (2014) reduced the FIS problem to finding a maximum weight clique in a graph and proposed an exact algorithm and several greedy heuristics. Zhou et al. (2014) dealt with the tactical problem with machine classes and spread-time constraints and developed a specialized branch-and-price algorithm. The tactical FIS problem is also related to the shift minimization personnel task scheduling problem, see Lin and Ying (2014), Smet et al. (2014).

There are three recent papers dealing with FIS problems under uncertain processing times using stochastic integer programming formulations. Branda et al. (2015) proposed a two-stage stochastic programming formulation which aims at maximizing the reward for processing the selected jobs and at the same time minimizing the costs for outsourcing additional machines. An operational problem devoted to optimal scheduling of all jobs to available machines was considered by Branda et al. (2016), who proposed an extended robust coloring reformulation of the FIS problem with random delays and suggested a tabu search algorithm for solving larger instances. Branda and Hájek (2017) observed that a network-based formulation enables us to optimally solve a larger instance by the mixed-integer programming solver IBM Cplex. Moreover, this formulation enabled the consideration of heterogeneous machines and job delays dependent on a selected machine.

The basic stochastic programming approach assumes the full knowledge of the probability distribution of random parts. However, in many real-life stochastic optimization problems, the probability distribution is not known precisely and the decision-maker must rely on its approximation. Therefore, it is necessary to investigate stability and robustness of the obtained solutions with respect to the choice of the distribution. Since the seminal work of Žáčková (1966), the minimax approach to robustness has obtained high attention in the stochastic programming literature. In this approach, the decision-maker hedges against the worst-case (the most pessimistic) distribution taken from a set of probability distributions called the ambiguity set. Previous works on distributionally robust scheduling employed moment conditions where the first two moments are given, see, e.g., Chang et al. (2017), Wiesemann et al. (2012). This paper proposes the first attempt to deal with distributionally robust FIS problems, moreover under a cardinality constrained ambiguity set. This set contains all distributions of delays where the joint distribution follows an Archimedean copula whereas a fraction of the marginals is stressed and the rest are left unchanged.

We deal with robust operational and tactical problems under ambiguous sets of probability distributions. We combine several results valid for the robust combinatorial problems and the network flow problems from Bertsimas and Sim (2003), who considered the cardinality constrained uncertainty set to deal with robustness with respect to the uncertain coefficients in the objective function. This uncertainty set was also considered by Bertsimas and Sim (2004), who gave a probabilistic interpretation for this choice as well. Since we deal with probabilistic distributions, we will use the more appropriate term of a cardinality constrained ambiguity set. In our case, the cardinality constrained ambiguity set enables us to look into the robustness and stability of the optimal values and solutions with respect to perturbations of the marginal distributions.

The maximization of the worst-case probability in the operational FIS leads to a minimax problem which is directly intractable. However, under our assumptions, we are able to find its tractable reformulation which leads to a cost–Conditional Value at Risk (CVaR) minimization under network flow constraints. This reformulation can be solved directly by a mixed-integer programming solver. Since this approach is already demanding, we propose a decomposition algorithm. We will obtain a second-stage (slave) problem which can be solved as a network flow problem with relaxed binary variables. The master problem can then be solved by the evaluation algorithm introduced by Bertsimas and Sim (2003) or, as we propose, by the golden-section search method, cf. Bazaraa et al. (2006). These approaches to the solution are then compared in the empirical study showing a significant improvement in the computational time using the new method.

The robust tactical problem belongs to a general class of chance constrained problems under ambiguity. In this area, mainly the moment conditions were considered to define the ambiguity set, see, e.g., Hanasusanto et al. (2015), Zymler et al. (2013). We will benefit from a special structure of the tactical FIS problem under the cardinality constrained ambiguity set. We propose a modified binary search over the number of machines where we solve an operational problem in each of the iterations. Moreover, we will investigate the influence of the copula dependence on the optimal number of machines in the numerical part.

This paper is organized as follows. In Section 2, we provide mathematical formulations of the distributionally robust FIS problems and formulate the distributional assumptions. In Section 3, we propose the reformulation of the robust operational problem and discuss the algorithms based on its decomposition. An algorithm for solving the robust tactical problem is then introduced in Section 4. In Section 5, we provide an extensive numerical study based on simulated instances. Section 6 concludes the paper.

2. Stochastic programming formulations

In this section, we propose formulation of the distributionally robust FIS problems and discuss the distributional assumptions. We will consider m parallel identical machines and n jobs. For each job j there is a known starting time s_j. We denote by $\mathcal{S} = \{s_1, \dots, s_n\}$ the set of all starting times when it is necessary to verify that at most one job is assigned to each machine, cf. Kroon et al. (1995, 1997). We assume that the finishing time is uncertain and it can be written as $f_j(\xi) = f_j + D_j(\xi)$, where f_j is a prescribed finishing time of the job j and $D_j(\xi)$ denotes a random delay which is a nonnegative random variable on probability space $(\Xi, \mathcal{A}, P)$, where Ξ is the set of elementary events, $\mathcal{A}$ denotes a $\sigma-$algebra and P is a probability measure. However, we assume that the probability measure P is not known precisely but it belongs to an ambiguity set $\mathcal{P}$.

The robust operational FIS can be formulated as a minimax problem where the worst-case probability that the schedule remains feasible, i.e., there is at most one job assigned to each machine at all starting times, is maximized:

$$\max_{x_{ji}} \min_{P \in \mathcal{P}} P\left(\xi \in \Xi : \sum_{\{j:\, s_j \le t < f_j(\xi)\}} x_{ji} \le 1,\ t \in \mathcal{S},\ i = 1, \dots, m \right) \tag{1}$$

$$\sum_{\{j:\, s_j \le t < f_j\}} x_{ji} \le 1, \quad i = 1, \dots, m, \quad t \in \mathcal{S}, \tag{2}$$

$$\sum_{i=1}^{m} x_{ji} = 1, \quad j = 1, \dots, n, \tag{3}$$

$$x_{ji} \in \{0, 1\}, \quad i = 1, \dots, m, \quad j = 1, \dots, n, \tag{4}$$