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Finding exact solutions for the Geometric Firefighter Problem in practice

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ABSTRACT

In the Geometric Firefighter Problem (GFP), one aims to maximize the total area shielded from a fire that radiates from a point inside a polygonal region, by constructing a subset of a given set of barriers. To decide which barriers to construct, a solution must take into account the speed of the circularly spreading fire and the barriers construction speed. A barrier is considered successfully constructed if the fire does not burn any still unconstructed point of the barrier. In this work, we consider the case where the initial set of barriers is comprised of rectilinear chords of the polygon. We present an Integer Programming (IP) model employed to solve the GFP to optimality along with procedures for preprocessing the instances, including primal algorithms and methods to reduce the problem size, as these constitute an essential step for solving harder instances. Moreover, we report on extensive experimental results that show that our IP model is an order of magnitude faster than the previous state-of-the art algorithm for the GFP. To further strain our algorithms, we introduce a new set of instances based on US national forests, which proved to be noticeably harder to solve than the previously available benchmark. An extended report on our experimental findings is presented along with a discussion that includes a restricted case where the constructed barriers must have pairwise disjoint interiors.

1. Introduction

According to a 2015 report from the United States Forest Service (USFS) (United States Forest Service, 2015) the percentage of the Forest Service's budget devoted to wildfire suppression rose from 16% in 1995 to more than 50% in 2015, and is predicted to increase to $1.8 billion by 2025. In 2014, the ten largest forest fires cost more than $320 million to the Forest Service. With so much investment required for fire control, techniques to improve this process are most wanted. In this work, we present algorithms and experimental results on a wildfire related problem, usually called the *Geometric Firefighter Problem* (GFP). This problem was recently proposed by Klein et al. (2014), who also proved its $\mathbb{NP}$-hardness. In the GFP, we are given a polygon describing the boundary of a fire-prone area one wants to protect from a fire, starting somewhere inside this polygon, regions whose total area is as large as possible, see Fig. 1 (right). This is to be accomplished by constructing (fire-proof) barriers selected from a set given as input along with the fire spreading speed and the construction speed of the barriers.

The GFP is a natural geometric extension of a widely studied graph problem, the Firefighter Problem (FP). In the FP, one is given a graph $G = (V, E)$, a set $V' \subset V$ of vertices on fire and a maximum number of firefighters. Through iterations over a discrete set of ticks starting at t_0, when only the vertices in V' are burning, each firefighter can choose to protect a single vertex not yet burned at each tick t_i, $i > 0$. The fire then spreads to all unprotected vertices adjacent to some vertex on fire. After a vertex starts burning or is protected by a firefighter, it remains in that state in all subsequent iterations. In this setting, many interesting questions may be asked: what is the maximum number of vertices that can be saved from the fire, or what is the minimum number of iterations required for the fire to stop spreading.

Next, we formalize the problem of interest.

1.1. The Geometric Firefighter Problem (GFP)

Let P be a simple polygon and B a set of continuous simple curves (barriers) of finite length interior to P, which can either be Jordan curves (plane curves homeomorphic to the unit circle) or have endpoints lying on the boundary of P. Let r be a point in the interior of P, v_f and v_b positive constants (speeds). Consider a fire that starts at r and circularly spreads at speed v_f. Any barrier in B can be constructed at speed v_b, i.e., in time proportional to its

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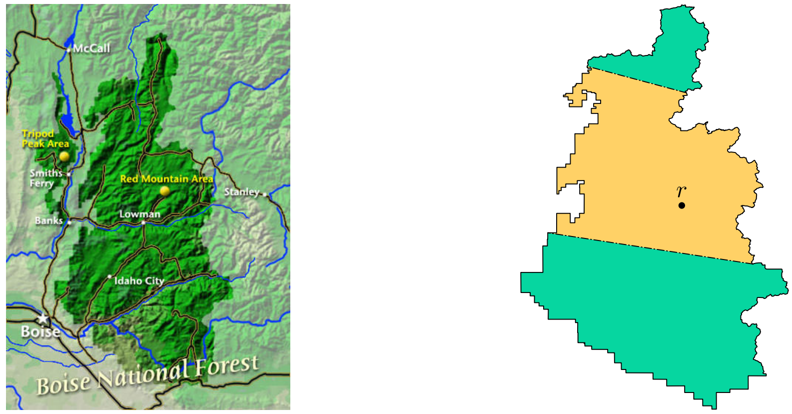

Boise National Forest.
Source: USFS.

Optimal solution of an instance generated with $v_f = 1$ and $v_b = 2$.

Fig. 1. Example of an instance based on an actual US national forest.

length. We say that a barrier b is *successfully constructed* if, during its construction, the fire never reaches any not yet constructed point, that is, for every point $p \in b$, $t_b(p) \geq t_f(p)$, where $t_b(p)$ and $t_f(p)$ are the time instants when p is constructed and reached by the fire, respectively. Therefore $t_f(p)$ depends on the total length of the barriers built up to that point. Since we assume that the fire does not surpass a successfully constructed barrier b, it is easy to see that b partitions P into two regions, one of which is *protected/shielded* from the fire by b. We wish to determine a subset of barriers from B that once successfully constructed, in a given order, maximizes the total area of P shielded from the fire. We require that once a barrier starts being built (in one of the two possible directions), its construction must be completed before another barrier can be started.

Fig. 1 shows a problem instance based on the Boise National Forest. The dashed segments in (b) comprise the set of constructed barriers in an optimal solution, selected from a set of 1500 randomly generated polygon diagonals. The fire source is located on the point labeled r, $v_f = 2$ and $v_b = 1$. The regions in green (■) correspond to the area shielded from the fire, while the orange (■) portion was consumed by the fire.

In general, unless multiple firefighters are allowed, we assume that once the construction of a barrier is started, building another one cannot commence until the previous one is complete. We also say that two intersecting segments/barriers *cross* whenever they share a point interior to both of them on which they are mutually transverse rather than mutually tangent.

1.2. Previous works

We first review the literature on the Firefighter Problem. Proposed in 1995 (Hartnell, 1995), during a conference talk, the Firefighter Problem has since attracted much attention from both graph theoretical and, more recently, experimental researchers. On the theoretical front, a great deal of work has been done towards proving bounds for the surviving rate of graphs of specific classes. The *surviving rate* $\rho(G, f, d)$ of a graph is defined as the average percentage of protected vertices considering fire break-outs on f vertices, with d available firefighters per round. Many classes of graphs have been studied with respect to their surviving rate, including trees (Cai and Wang, 2009; Costa et al., 2013), planar graphs (Kong et al., 2012; Wang et al., 2014; 2012), outerplanar graphs (Cai and Wang, 2009; Wang et al., 2011), digraphs (Kong et al., 2014) and grids (Gavenčiak et al., 2014). For infinite grids, bounds on the number of firefighters required to contain the fire have been determined (Develin and Hartke, 2007; Feldheim and Hod, 2013; Wang and Moeller, 2002). The FP is also known to be $\mathbb{NP}$-hard for general trees of maximum degree three (Finbow et al., 2007) and cubic graphs (King and MacGillivray, 2010).

On the experimental front, García-Martínez et al. (2015) report results comparing a number of Integer Programming techniques and heuristics over random general graphs and random geometric graphs. Also, there have been several works that propose new variants of the FP (Lipinski, 2017; Michalak, 2014; Michalak and Knowles, 2016) and that tackle them, as well as the original problem, using a variety of metaheuristics (Blum et al., 2014; Hu et al., 2015; Michalak, 2017a; 2017b). Notice that since in the FP the burning times can be discretely measured while in the GFP they are continuous, adapting those methods would not be straightforward. However, as we will see in Section 3.6, heuristics that work well for the FP include strategies useful for the GFP, such as defending areas under immediate threat.

The GFP was proposed in 2014 by Klein et al. (2014) who proved the $\mathbb{NP}$-hardness of the cases where P is a simple polygon and B is a set of diagonals of equal length, or P is a convex polygon and B is the set of all its diagonals, and when P is a star-shaped polygon and there are no restrictions over the set of barriers. The paper considers that the curves in B are compliant with a *linearity condition*. This condition states that barriers should not cooperate in order to protect a region, i.e., for any subset $B^* \subseteq B$, the union of the regions protected by the curves in B^* must be equal to the union of the regions protected individually by each barrier in B^*. In the same work, an ~11.65-approximation algorithm for the GFP is presented. A related $\mathbb{NP}$-hard problem, dubbed *Budget Fence Problem* is introduced, where one is asked to maximize the total area fenced from a contaminated region $R \subset P$, given a budget on the total barrier length. In this version, no construction speed is given nor a fire speed, and the constructed barriers must not cross. The authors also discuss a polynomial-time approximation scheme (PTAS) for this problem, which can be used for solving the GFP in the case where B includes no closed curves and the constructed barriers do not cross.

In a previous work (Zambon et al., 2016), we considered the GFP for simple polygons, with B comprising the set of all diagonals. We proposed the first Integer Programming (IP) model and the required preprocessing algorithms to solve the problem to optimality. We also reported on experimental results for a benchmark composed of instances with polygons of up to 300 vertices. That work represented the first attempt to address the GFP in practice.