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Reliable single allocation hub location problem under hub breakdowns

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ABSTRACT

The design of hub-and-spoke transport networks is a strategic planning problem, as the choice of hub locations has to remain unchanged for long time periods. However, strikes, disasters or traffic breakdown can lead to the unavailability of a hub for a short period of time. Therefore it is important to consider such events already in the planning phase, so that a proper reaction is possible; once a hub breaks down, an emergency plan has to be applied to handle the flows that were scheduled to be served by this hub. In this paper, we develop a two-stage formulation for the single allocation hub location problem which includes the reallocation of sources to a backup hub in case the hub breaks down. In contrast to related problem formulations from the literature, we keep the non-linear structure of the problem in our model. A branch-and-cut framework based on Benders decomposition is designed to solve large scale instances to proven optimality. Thanks to our decomposition strategy, we keep the structure of the resulting formulation similar to the classical single allocation hub location problem, which in turn allows to use classical linearization techniques from the literature. Our computational experiments show that this approach leads to a significant improvement in the performance when embedded into a standard mixed-integer programming solver. We report optimal solutions for instances much bigger than those solved so far in the literature.

1. Introduction

Hub location problems are strategic planning problems which have been studied for more than 30 years (Alumur and Kara, 2008; Campbell and O'Kelly, 2012; Klincewicz, 1991; O'Kelly, 1987). The problem consists of organizing the mutual exchange of flows between a large set of depots by choosing a set of hubs out of the set of possible locations and assigning each flow to a path from source to sink being processed at a small number of hubs in between. The aim is to utilize economies of scale in transportation: Although the transport routes are longer and additional costs for hubs apply, the savings from bundled transport usually outweigh these costs. The economies of scale are usually modeled as being proportional to the transport volume, defined by multiplication with a discount factor $\alpha \in [0, 1]$. The resulting trade-off has to be optimized. Typical applications of hub-based networks arise in airline (Jaillet et al., 1996), postal (Ernst and Krishnamoorthy, 1996), cargo (Taylor et al., 1995), telecommunication (Klincewicz, 1998) and public transportation (Nickel et al., 2001) services.

There are numerous variants of hub location problems. Depending on the way in which nodes may be assigned to hubs, hub location problems can be classified as either multiple allocation (Ernst and Krishnamoorthy, 1998) or single allocation (Ernst and Krishnamoorthy, 1996; O'Kelly, 1987) hub location problems. In multiple allocation problems, the flow of the same node can be routed through different hubs, while in single allocation problems, each node is assigned to exactly one hub. Thus, all flows originating or destining to a node have to be routed via the respective hub to whom it is assigned. Moreover, each of these problems can be classified as capacitated or uncapacitated depending on various types of capacity restrictions. In particular, there can be limitations on the total flow routed on a hub-hub link (Labbé et al., 2005) or on the volume of flow into the hub nodes (Ernst and Krishnamoorthy, 1999). As an extension of these models, in Correia et al. (2010), Contreras et al. (2012), and Rostami et al. (2016) the authors consider a version where individual capacity levels can be installed for each hub location. Accordingly, not only the hub nodes have to be chosen but also the capacity level at which each of them will operate.

O'Kelly (1987) proposed the first quadratic integer programming formulation for classic uncapacitated, single allocation p-hub median problem where the number of hubs, denoted by p, is given

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(the so-called p-Hub Median Problem). Since then, many exact and heuristic algorithms have been proposed in the literature, dealing with locating both a fixed and a variable number of hubs, e.g., by Campbell (1994), Ernst and Krishnamoorthy (1996), Skorin-Kapov et al. (1996), Contreras et al. (2011), Meier et al. (2016), and Ilić et al. (2010). Due to the quadratic nature of the problem, many attempts have been made to linearize the objective function so that the resulting lower bound is strong enough to be used in a branch-and-bound algorithm. Skorin-Kapov et al. (1996) and Ernst and Krishnamoorthy (1996) proposed "path based" and "flow based" Mixed Integer Linear Programming (MILP) formulations, respectively. For recent surveys on hub location problems we refer the reader to Alumur and Kara (2008) and Campbell and O'Kelly (2012).

From the perspective of reliability, the traditional hub location models lack one important property: The inclusion of uncertainty in hub operation. Earth quakes, flooding, strikes or accidents might be reasons for a breakdown of a hub. Although those deviations from the standard scenario are rare events, they have a major effect on the entire transportation network as flows get stuck on their normal routes. Since in practice it is of major importance to maintain a high service level, the network has to be recovered in order to maintain all flows.

The concept of reliability under breakdown uncertainty has received considerable attention in the context of the facility location problem in which locations in an one-directional transportation network have to be found in order to minimize the transportation cost. In the model of Snyder and Daskin (2005), the breakdown of multiple facilities is considered. Each costumer is assigned a primary facility and a set of ordered backup facilities to be chosen if the prior facilities fail. The authors solve their formulation by using Lagrangian relaxation within a branch-and-bound scheme. Cui et al. (2010) extend the model of Snyder and Daskin (2005) by location specific instead of homogeneous failure probabilities. A continuous approximation heuristic omitting location and assignment details is developed to find near-optimal solutions. Li et al. (2013) only consider scenarios where a single facility may fail, but they introduce fortification costs which can be payed for facilities in order to reduce their failure probabilities. Again a Langrangian relaxation is used to solve this problem. Furthermore, Álvarez-Miranda et al. (2015) introduce the concept of robust recovery proposed by Liebchen et al. (2009) to facility location with breakdowns, where after the scenario is revealed, restrict action can be taken to make the solution feasible again. Álvarez-Miranda et al. (2015) apply this concept to a facility location problem, where the set of facilities, customers, and in-between connections may change when a scenario is revealed. Then a restricted action recovers feasibility. The authors apply a Benders decomposition approach to solve their recoverable robust problem formulation.

Even though the deterministic hub location problems have been well-studied over the years, the literature addressing hub breakdowns is rather limited. Strategies to recover transportation networks in case of partially or completely unavailable hubs were studied by Løve et al. (2002) and O'Kelly et al. (2006) in the context of air transportation and telecommunication systems, respectively. However, these policies have to respond to an initially planned transportation networks, typically without consideration of hub breakdowns. Hence, flows have to be rerouted via other hubs by potentially significantly larger costs than in the initial scenario.

Further models of hub breakdowns in hub location problems were studied for the multiple allocation variant. Chaharsooghi et al. (2017) either reassign flows to other hubs or penalize them for not being in service in case of a hub breakdown. They solve instances on up to 80 nodes by a neighborhood search heuristic. The model of Kim (2012) represents a problem variant, where the backup hubs are distinct from the original hub and are only used in case a original hub fails. An 18-node instance is solved to optimality. Lei (2013) examines the impact of hub breakdown in an existing transportation network. The goal is to identify the subset of r breakdown hubs that leads to the most severe degradation in transportation cost. Instances on up to 40 nodes are solved to optimality.

For the single allocation hub location problem, the impact of hub breakdowns to a transportation network is considered within the strategic planning phase by An et al. (2015), Azizi et al. (2014), and Tran et al. (2015). An et al. (2015) hedge against hub breakdown by rerouting each shipment affected by a closed hub. Note that nodes then could potentially be allocated to more than one hub, in contrast to the usual setting of single allocation hub location problems. The authors could solve instances on up to 25 nodes using Lagrangian relaxation. Azizi et al. (2014) determine for each hub a backup hub which completely takes over the flows of the closed hub. Instances on up to 10 nodes are solved exactly and instances on up to 80 nodes heuristically by a genetic algorithm. Tran et al. (2015) assume that multiple hub can simultaneously fail. For each hub a sequence of backup hubs is determined, where a backup hub takes over all flow from the original hub, when the original hub and all previous backup hubs break down. Their model is linearized with help of a probability lattice calculating the probability that a path is used in the transportation network. Instances up to 20 nodes are solved to optimality with a commercial solver. Further the author present a tabu search to solve the problem heuristically.

However, from a mathematical modeling point of view, all three approaches by An et al. (2015), Azizi et al. (2014), and Tran et al. (2015) neglect the quadratic structure of the problem by considering a path based formulation Skorin-Kapov et al. (1996) for single allocation hub location problems. This blows up the problem under loss of valuable information and leads to intractable models for large scale instances. Following An et al. (2015), Azizi et al. (2014), and Tran et al. (2015), our aim is also to include hub breakdowns in the strategic planning of a hub-based transportation network with single allocations. Therefore, the objective in our proposed model includes the expected additional transportation costs caused by a hub breakdown in the classical, i.e. breakdown-free, scenario with the costs in the breakdown scenarios. Following Liebchen et al. (2009) in the concept of recoverable robustness, whenever a breakdown scenario is revealed, the transportation links to the closed hub have to be rerouted via another operating hub, called backup hub, in order to maintain a high service-level.

1.1. Our contribution

In order to include recovery decisions in a mathematical model, we extend the classical quadratic formulation O'Kelly (1987) for single allocation hub location problem by incorporating the reallocation variables into the mathematical model. Conversely to the previous models (An et al., 2015; Azizi et al., 2014; Tran et al., 2015) for reliable single allocation hub location problems, where a path-based formulation Skorin-Kapov et al. (1996) is used to linearize the quadratic term to compute the inter-hub transportation costs, the inherent quadratic structure of the classical formulation is kept in our Mixed Integer Quadratically Constrained Quadratic Program (MIQCQP). This allows us to preserve valuable problem structure for our solution method.

To solve the proposed model, we develop a two stage decomposition base approach where the first stage solves a breakdown-free scenario while the second stage problem reacts to hub breakdowns by rerouting affected transportation arcs. Similar to